Unit-4

LASERS:

Introductory Concepts

Objective

The objective of this unit is to give the student introductory but fundamental concept regarding lasers. These concepts include the fundamental difference between spontaneous and stimulated emission, their origins, and basic requirements of laser emission, e.g., active medium, population inversion, and optical feedback. A brief history is also given for general interest.

4.1 Introduction and Brief History

Laser is a device that amplifies light and produces a highly directional, high intensity beam that typically has an almost pure frequency or wavelength. It comes in sizes ranging from approximately one-tenth the diameter of a human hair to the size of a very large building, in powers ranging from 10^{-9} to 10^{20} watts, and in wavelengths ranging from the microwave to the soft x-ray spectral regions with corresponding frequencies from 10^{11} to 10^{17} Hertz. Lasers have pulse energies as high as 10^{4} joules and pulse duration as short as 10^{-15} seconds.

Lasers are key components of some of our most modern communication systems and are the “phonographic needle” of compact disc players. They are used for heat treatment of high-strength materials, such as the pistons of automobile engines, and provide a special surgical knife for many types of medical procedures. They act as target designators for military weapons and are used in the checkout scanners. They can easily drill holes in the most durable of materials and can weld detached retinas within the human eye.

The word laser is an acronym for Light Amplification by Stimulated Emission of Radiation. One of the major processes in lasers is stimulated emission. Albert Einstein gave the idea of stimulated emission of radiation in 1917. Until that time, physicists had believed that a photon could interact with an atom in only two ways: it could absorbed and raise the atom to higher energy level or be emitted as the atom is dropped to a lower energy level. Einstein proposed a third possibility that a photon with energy corresponding to that of an energy level transition could stimulate the atom in the upper level to drop to the lower level by emission of another photon with the same energy and phase as the first one.

The stimulated emission is least known because at thermodynamic equilibrium more atoms are in lower energy levels than in higher ones. Thus a photon is much more likely to encounter an atom in a lower level and be absorbed than to encounter one in a higher level and stimulate emission.

The first efforts to use the idea of stimulated emission were a few decades latter in the microwave region and led to the invention of maser “microwave amplification by stimulated emission of radiation”. The maser concept evolved nearly simultaneously in the United States and Soviet Union. A physicist Charles H. Townes of the Colombia University is known as the inventor of maser.

Townes thought that molecules in the excited state could be stimulated to emit microwaves when placed in a special resonant cavity designed to enhance the emission. He outlined the idea to post-doctoral fellow Herbert Zeiger and graduate student James P. Gordon in 1951, and by 1953 they had a working maser. Meanwhile, Alexander M. Prokhorov and Nikolai Basov (1954) of Lebedev Physics Institute in Moscow calculated the details of maser action and published shortly after Townes’s results. The contributions of all these three men were recognised and they were awarded Nobel Prize in physics in 1964.

After maser, Townes and other physicists began looking beyond the microwave region to shorter wavelengths. They realised that at those wavelengths the physical conditions required to produce stimulated emission would be very different. Townes and Arthur L. Schawlow (of Bell Labs.) worked out many key parameters. Their results were published in a major paper in 1958 and they also filed a patent application before the paper was published.

Meanwhile, a graduate student at Columbia, Gordon Gould, was working out his own analysis of the conditions required for stimulated emission at visible wavelengths. Gould wrote his proposals in a set of notebooks in 1957 but did not try to publish his results promptly. He wanted to patent his work but due to some bad legal advice, he did not file a patent application until 1959, about nine months after the Schawlow-Townes patents application was submitted. The Schawlow-Townes patent was granted promptly, but Gould's application ran into a lengthy process. Finally, Gould got four patents in 1977, 1979, 1987 and 1988, based on divisions of his original application.

Schawlow and Townes have received many scientific honours for their work, but Gould received little recognition until his patents were issued. Although Gould loses the prestige race, yet he benefited financially. However, it was he, who first coined the word LASER in his notebooks. Schawlow and Townes described their idea as an "optical maser".

Publication of Schawlow-Townes paper stimulated many scientists to build lasers, and interest spread beyond the narrow scientific community. Schawlow, Gould and most researchers thought that gases were the best materials for lasers. However, Theodore H. Maiman, a young scientist at Hughes Research Laboratories in Malibu, California, quietly disagreed. He preferred synthetic ruby crystals to gases, although some theorists insisted that ruby would not work. Maiman, who had studied energy levels in ruby extensively, proved that the theorists were wrong. In mid-1960 he proudly demonstrated the world's first laser, the ruby laser. The laser era was born.

Maiman had to face a lot of difficulties; Hughes management told him to stop work on the ruby laser, the prestigious journal, Physical Review Letters, rejected his report of ruby laser but it was published in Nature. Today, however, Maiman is universally recognised as the person who built first laser and has received a number of honours.

Maiman's demonstration of the ruby laser opened the floodgates and was followed by the demonstration of helium neon laser by Ali Javan, W.R. Bennett Jr., and Donald R. Harriot at Bell Telephone Laboratories in Murray Hill, New Jersey. Their first helium-neon laser operated at 1.15 micrometers (μm) in the near infrared. Latter, other researchers found the 632.8 nanometer (nm) red line which made the helium-neon laser most popular.

The laser boom really got going. In 1961, L. F. Johnson and K. Nassau demonstrated the first solid-state neodymium laser in which the Nd ions were dopant in calcium tungstate, but the today's best choice of neodymium host for best commercial applications- yttrium aluminium garnet (YAG), was demonstrated as a laser material in 1964. Three separate groups demonstrated semiconductor diode lasers nearly simultaneously in fall 1962. All the teams demonstrated the same gallium arsenide diodes cooled to the 77 K temperature of liquid nitrogen and pulsed with high-current pulses lasting a few microseconds. The next few years saw the birth of several more important lasers. W. B. Bridges (1964) observed 10 laser transitions in the blue and green parts of the spectrum from singly ionised argon. C. Kumar and N. Patel (1964) obtained a 10.6 μm laser emission from carbon dioxide. Sorokin and J. R. Lankard (1966) demonstrated the first organic dye laser; hydrogen chloride emitting at 3.7 μm was demonstrated in 1965 by J. V. V. Kaspar and G. C. Pimentel.

4.2 Basic Principle of Lasers

In order for most lasers to operate, three basic conditions must be satisfied. First, there must be an active medium, that is, a collection of atoms, molecules, or ions that emit radiation in the optical part of the electromagnetic spectrum. Second, a condition known as population inversion, i.e., more number of atoms available at the higher energy level as compare to lower energy level, must exist. This condition is highly abnormal in nature. It is created in a laser by an excitation process known as pumping. Finally, for true laser oscillation to take place there must be some form of optical feedback present in the laser system. If this was not present, the laser might serve as an amplifier of narrow-band light, but it could never produce the highly collimated, monochromatic beam that makes the laser so useful.

4.2.1 Active Medium

The heart of a laser system is a material capable of emitting radiation of the required energy. This material, known as active medium, may have any form e.g. solid, liquid, or gas but must contain a set of energy levels in which it can absorb or emit energy in the form of optical radiation.

4.2.1.1 Energy levels

In the macroscopic world, energy might seem to vary continuously, like the level of sand in a pail. If we look closely at the sand, we can see that it is made up of many separate grains, and that we can add or subtract only one grain at a time. Likewise, atoms and molecules can only have certain amounts of energy. We call these energy states or levels.

For example, let's look at the simplest atom (hydrogen) in which a single electron circles a nucleus that contains a single proton. At first glance, the hydrogen atom looks like a very simple solar system, with a single planet (the electron) orbiting a star (the proton). The force that makes the atom stable is the attraction between the positive charge of the proton and the negative charge of the electron.

The electron can occupy only certain orbits, as shown in Figure 4.1. We show the orbits as circles for simplicity, but we can't really measure exactly what the orbit looks like. If we add energy to our simple planetary system, the planet would move farther from the star. The same happens in the hydrogen atom. As we add more energy to the atom, the electron moves to more and more distant orbits. However, there is crucial difference between the behaviour of a hydrogen atom and our imaginary planetary system. The planet can be at any distance from the sun, or could even fall into it. However, the electron can only occupy certain orbits also called energy levels. These energy levels are plotted in Figure 4.1. Atom is, in general, lies in the lowest possible energy level, the ground state.

The energy levels in the hydrogen atom get closer together as they get higher above the ground state. Eventually the differences become vanishingly small. If the electron gets too much energy, it escapes from the atom altogether, a process called ionization. If we define the energy of the ionized hydrogen atom to be zero, we can write the energy of the atom E as a negative number using the simple formula:

$$E = -R/n^2 \tag{4.1}$$

where, R is a constant (2.178×10^{-18} Joules), n is the quantum number of the orbit (counting outwards, with one the innermost level).

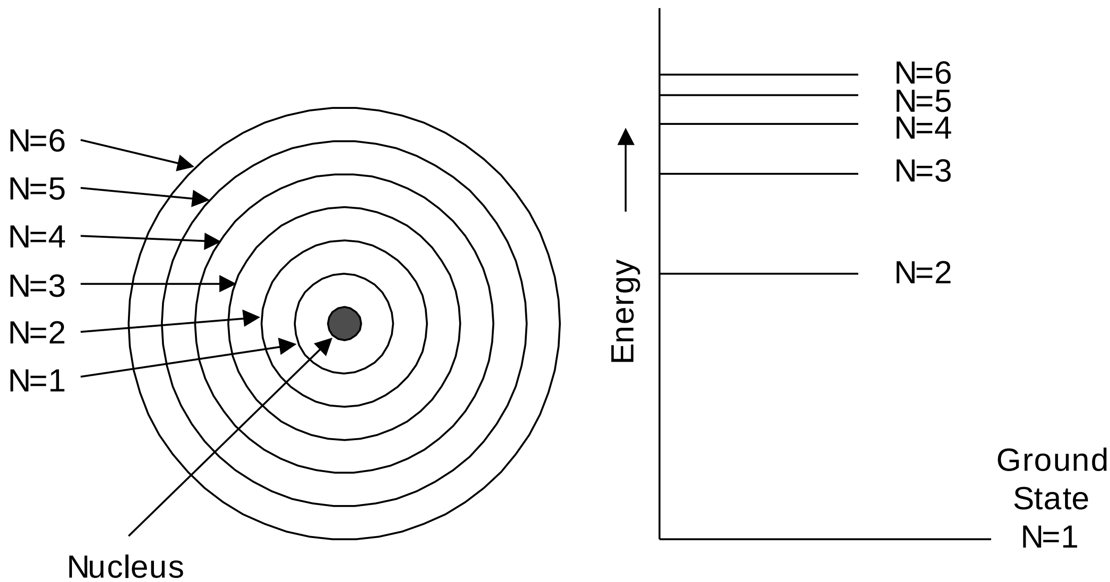


Figure 4.1 The hydrogen atom and the corresponding energy levels.

4.2.1.2 Transitions

The process of making changes or "transitions" between energy levels is very important to laser physics. To look at the process, let's start again with the hydrogen atom. The electron needs to gain energy to move from ground state to higher energy level. Conversely, it must release energy when it drops from a higher level to lower one. One of the most convenient ways (although by no means the only way) for an electron to absorb or release energy is as a photon, a quantum of electromagnetic energy. The photon energy equals the difference in energy between the two energy levels.

Suppose we start with the electron in the ground state and want to raise it up one step to the first excited level. To do so, we must give the electron exactly as much extra energy as the difference in energy between the ground state and the first excited level. Conversely, for the electron to drop from the first excited level to the ground state, it must emit a photon with as much energy as the difference in energy between the two levels. In short, the photon energy equals the transition energy. The energy of a photon can be represented by $h\nu$; in the frequency term, where h (= 6.626×10^{-34} J.s) is the Plank's constant and ν is the frequency of the transition, or in the wavelength form hc/λ, where c is the velocity of light in vacuum and λ is the wavelength of the photon. An electron in a hydrogen atom can emit or absorb light at only certain wavelengths.

This neat ordering of energy levels is evident only in hydrogen atom, where there is just a single electron. Add more electrons, and the energy level picture quickly becomes more complicated. Electrons interact with each other, and with the nucleus, shifting energy levels slightly. Electrons can occupy subshells within each shell. The more complex the energy level structure, the more transitions between energy levels is possible. The more transitions, the larger number of possible spectral lines. Superimpose them all on a single spectrum and look almost like a set of random lines.

Another complication is that all transitions are not equally likely. One reason is that more atoms are in some states than in others. For example, under normal circumstances, more atoms are in ground state than in excited levels. Another is that quantum mechanical rules make some transitions much more likely than others. That means that an atomic or molecular species will absorb or emit some wavelengths much more strongly than others. This effect shows up both in absorption spectra (which shows that wavelengths the material absorbs when light from another source passes through it) and emission spectra (the wavelengths the material emits when it is itself de-excited).

4.2.1.3 Types of Transitions

So far we have concentrated on electronic transitions, partly because we picked the hydrogen atom as our introductory example. Electronic transitions can cover a wide range of wavelengths. These occur at ultraviolet, visible, or infrared wavelengths from 100 nm in the ultraviolet through to near infrared wavelengths.

The shortest wavelengths come from inner-shell electronic transitions in heavy elements, which involve much more energy than outer-shell transitions. These short wavelengths are considered to be X-rays. On the other hand, transitions between high lying electronic energy levels (say, level 18 and 19 of hydrogen) involve very little energy, putting them deep in the infrared, microwave, or even radio-frequency range. Because these are qualitatively different than higher-energy transitions of outer electrons, they are put into special class called

Rydberg transitions.

Neither Rydberg transitions nor X-ray emission falls into the optical region, which is a very small portion of the electromagnetic spectrum, therefore, these are not likely events under normal laser conditions. Transitions between nuclear energy levels can produce even higher energy photon, called gamma rays.

On the other end of the wavelength spectrum are transitions between vibrational and rotational energy levels of molecules. Vibrational transition energies typically correspond to wavelengths of a few to tens of micrometers; rotational transitions have less energy, typically corresponding to wavelengths of at least 100 micrometers.

Transitions in two or more types of energy levels can occur simultaneously. For example, a molecule can undergo a vibrational and rotational transition simultaneously, with the resulting wavelength close to that of more energetic vibrational transition. Many infrared lasers emit families of closely spaced wavelengths on such vibrational-rotational transitions.

Remember in considering transitions that longer wavelengths correspond to lower energy, and shorter wavelengths correspond to higher energy ($E = hc/\lambda$). Thus, the energy of a vibrational transition is much larger than that of a rotational transition, even though the rotational wavelength is much larger. A combination of a rotational and vibrational transition thus has only slightly different energy than the original vibrational transition. Transition energies or frequencies add together in a straightforward manner:

$$E_{1+2} = E_1 + E_2 \tag{4.2}$$

where, E_{1+2} is the combined transition energy, E_1 and E_2 are the energies of the separate transitions. The same rule holds for frequencies, with ν substituted for the energy. However, wavelength (λ) of combined transitions add by an inverse rule:

$$1/\lambda_{1+2} = 1/\lambda_1 + 1/\lambda_2 \tag{4.3a}$$

or $$\lambda_{1+2} = 1/(1/\lambda_1 + 1/\lambda_2) \tag{4.3b}$$

4.2.1.4 Spontaneous Emission

We know that atoms have well defined energy levels and they can be pushed to the higher energy states (excited states) by absorption of a photon or by some other means e.g. electric discharge. Atoms spent some time in the excited state and then decay to the lower energy state. The average time required to de-exite the 1/e number of atoms from the upper level to the lower level is called lifetime of the state. This lifetime can be very small e.g. in nano-seconds and up to a few seconds. Typically, for electronic transitions it is in the order of ten nano-seconds. After spending this period in the excited state, atoms come down to the lower energy state by emitting a photon. This emission of photon from an excited atom is called spontaneous emission. This phenomenon one sees in daily life. Sun, bulb, tubelight and all the fluorescent devices emit photons spontaneously in the visible region. These photons are emitted in all the direction and illuminate the whole area, probably, this is the blessing of spontaneous emission.

Example 4.1*: Describe spontaneous emission from excited helium*

When we see helium discharge lamp directly, it appears to emit pinkish white light as shown in Figure 4.2. In fact in the discharge, helium atoms are being excited in the upper energy levels. They soon give up their energies by dropping down to lower energy level with emission of photons. This spontaneous emission involves quite a number of different energy levels and thus produces the photons of different colours. When viewed the same discharge through a diffraction grating (an optical element to

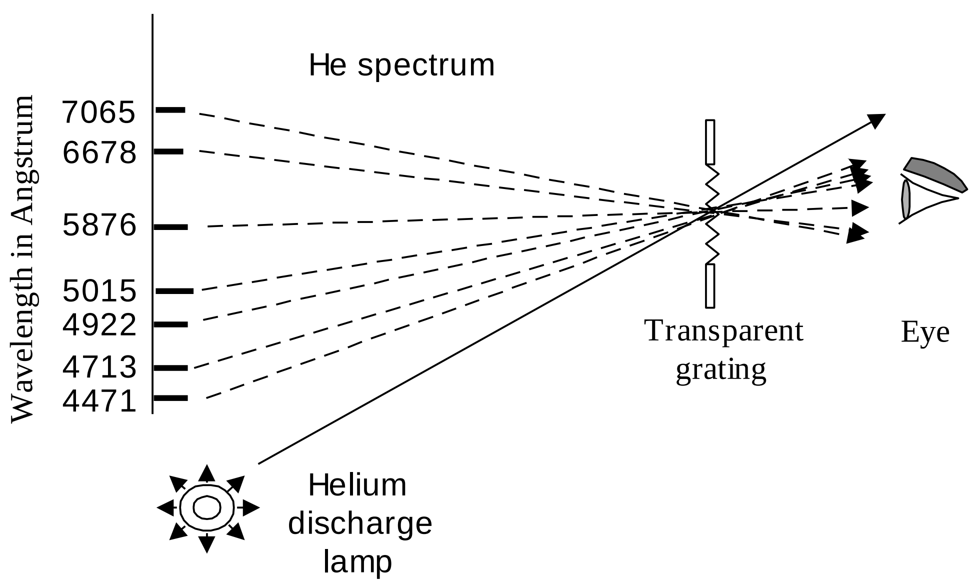


Figure 4.2 Helium discharge spectrum observed through a transmission grating.

separate different wavelengths of light) multiple images of different colours of the lamp appear at different angles. A strong yellow line at 588 nm is prominent but violet, green, blue, red and deep red lines can also be seen.

To understand the spontaneous emission a little more, let us consider two energy levels, 1 and 2, of some given material, their energies being E_1 and E_2 ($E_1<E_2$) and population densities N_1 and N_2 as shown in Figure 4.3. It is convenient, however, to take level 1 to be the ground level. Let us now assume that an atom (or molecule) of the material is initially in level 2. Since $E_2>E_1$, the atom will tend to decay to level 1. The atom must therefore release the energy, corresponding energy difference (E_2-E_1). This energy is delivered in the form of a photon of frequency ν given by

$$\nu = \frac{(E_2 - E_1)}{h} \tag{4.4}$$

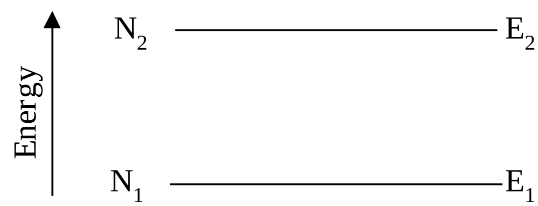


Figure 4.3 Two level energy system.

Spontaneous emission is therefore characterized by the emission of a photon of energy $h\nu = E_2-E_1$, when the atom decays from level 2 to level 1 (Figure. 4a). Note that radiative emission is just one of the two possible ways for the atom to decay. The decay can also occur in a non-radiative way. In this case the energy difference E_2-E_1 is delivered in some form other than a photon (e.g., it may go into kinetic energy of the surrounding molecules).

The probability of spontaneous emission can be characterised in the following way: Let us suppose that, at time t, there are N_2 atoms (per unit volume) in level 2. The rate of decay of these atoms due to spontaneous emission, i.e., $(dN_2/dt)_{sp}$, will obviously be proportional to N_2. We can therefore write

$$\left(\frac{dN_2}{dt}\right)_{sp} = -A_{21}N_2 \tag{4.5}$$

The coefficient A_{21} is called the spontaneous emission probability or the Einstein coefficient. The quantity $\tau_{sp}=1/A_{21}$ is called the spontaneous emission lifetime. The numerical value of A_{21} (and τ_{sp}) depends on the particular transition involved.

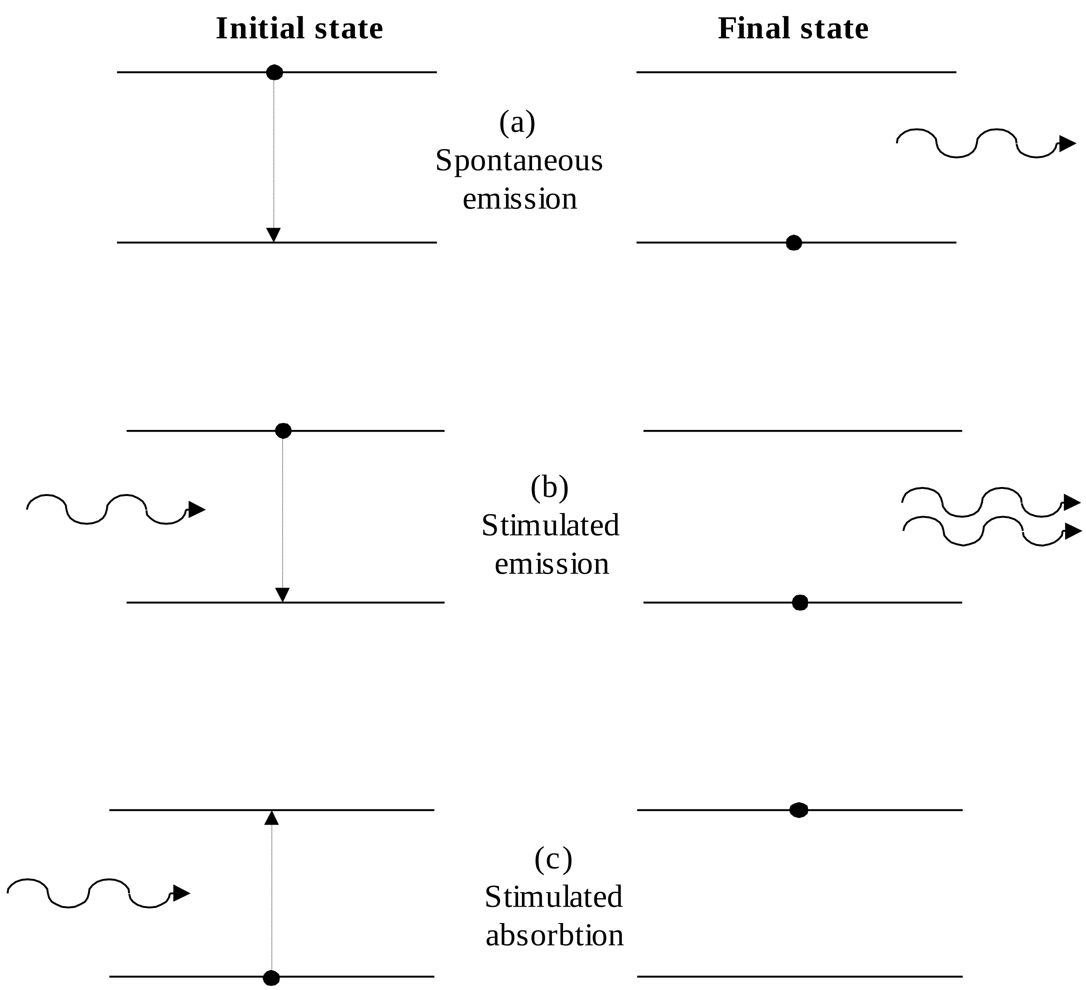


Figure 4.4 Energy level diagram illustrating (a) spontaneous emission, (b) stimulated emission and (c) stimulated absorption.

In spontaneous emission each individual atom acts like a small randomly oscillating antenna emitting at the transition frequency. Therefore, the total emission from a collection of spontaneously emitting atoms exhibit noise-like character (Figure 4.5).

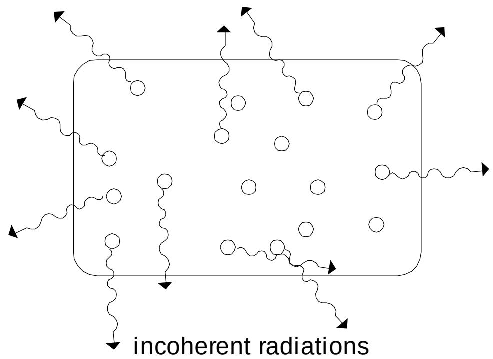


Figure 4.5 Spontaneous emission is incoherent or noise-like character, emerging randomly in all directions.

4.2.1.5 Stimulated Emission

In stimulated emission, the atom is triggered to undergo the transition in the presence of photons of energy (E_2-E_1). In these transitions each individual atom acts like a passive resonant antenna that is set oscillating by the applied signal. Therefore, the internal motion or oscillation in the atom is not random, but is driven by an applied signal. Let us again assume that the atom is found initially in level 2 and a photon of frequency ν as of equation (1) is incident on the material. Since this wave has the same frequency as the atomic frequency, there is a finite probability that this wave will force the atom to undergo the transition $2 \rightarrow 1$. In this case the energy difference E_2-E_1 is delivered in the form of a photon that adds to the incident one as shown in Figure 4b. There is, however, a fundamental distinction between the spontaneous and stimulated processes. In the case of spontaneous emission, the atom emits a photon that has no definite phase relation with that emitted by another atom. Furthermore, the photon can be emitted in any direction. In the case of stimulated emission, since the process is forced by the incident photon, the emission of any photon adds in phase to that of the incoming wave. This wave also determines the direction of the emitted wave. In this case, too, we can characterise the process by means of the equation

$$\left(\frac{dN_2}{dt}\right)_{st} = -\rho_\nu B_{21} N_2 \tag{4.6}$$

where $(dN_2/dt)_{st}$ is the rate at which transitions $2 \rightarrow 1$ occur as a result of stimulated emission, B_{21} is called the Einstein coefficient for stimulated emission, the energy density $\rho_\nu = Nh\nu$, where N is the number of photons per unit volume having frequency ν.

4.2.1.6 Absorption

All the atoms and molecules have discrete energy levels. An atom can absorb a photon and become excited. Let us now assume that the atom is initially in level 1. If this is the ground level, the atom will remain in this level unless some external stimulus is applied to it. We shall assume that a photon of frequency ν given by equation (1) is incident on the material. In this case there is a finite probability that the atom will be raised to level 2 (Figure 4.4c). The energy difference E_2-E_1 required by the atom to undergo the transition be obtained from the energy of the incident photon. This is the absorption process.

In a similar fashion to equations 2 & 3, we can write an equation for absorption rate,

$$\frac{dN_1}{dt} = -\rho_\nu B_{12} N_1 \tag{4.7}$$

where N_1 is the number of atoms per unit volume that at the given time are lying in level 1 and B_{12} is called the Einstein coefficient for stimulated absorption.

In the absorption process, the incident photon is simply absorbed to produce the $1 \rightarrow 2$ transition. In the beginning of the century, Einstein also showed that the coefficients of stimulated emission and absorption are equal, i.e. $B_{21}=B_{12}$ (assuming that degeneracy in both the upper and lower level is same)

4.2.1.7 The Einstein Relations

Einstein showed that the parameters describing the above three processes are related through the requirement that for a system in thermal equilibrium, the rate of upward transitions (E_2 to E_1) must be equal to the rate of downward transition processes.

We can write the upward transition rate as $N_1\rho_\nu B_{12}$. The total downward transition rate is the sum of the induced and spontaneous contributions i.e. $N_2\rho_\nu B_{21}+N_2A_{21}$. In the preceding discussions A_{21}, B_{21} and B_{12} are called the Einstein coefficients. The relationship between them can be established as follows.

For a system in equilibrium, the upward and downward transition rates must be equal and hence we have

$$N_1\rho_\nu B_{12} = N_2\rho_\nu B_{21}+N_2A_{21} \tag{4.8}$$

Thus

$$\rho_\nu = \frac{N_2A_{21}}{N_1B_{12} - N_2B_{21}} \tag{4.9a}$$

or

$$\rho_\nu = \frac{A_{21}/B_{21}}{N_1/N_2 - 1} \tag{4.9b}$$

The populations of various energy levels of a system in thermal equilibrium are given by Boltzmann statistics to be:

$$N_j = \frac{N_0 \exp(-E_j/kT)}{\sum_i \exp(-E_i/kT)} \tag{4.10}$$

where N_j is the number of atoms in the jth level with energy E_j, N_0 is the total number of atoms, k is Boltzmann constant, and T is temperature in Kelvin.

Hence
$$\begin{aligned} \frac{N_1}{N_2} &= \exp((E_2 - E_1)/kT) \\ &= \exp(h\nu/kT) \end{aligned} \tag{4.11}$$

From equations 4.9b and 4.11 we get ratio of spontaneous emission to stimulated emission in thermodynamic equilibrium as

$$\frac{A_{21}}{\rho_\nu B_{21}} = \exp(h\nu / kT) - 1 \tag{4.12}$$

Equation (4.9) shows that whenever there is emission of photons, each in thermal equilibrium, stimulated emission is always present. ;However, the ratio of stimulated emission to spontaneous emission depends on the temperature (in thermal equilibrium) frequency of the photons and the number of photons available for stimulation.

Example 4.2: *Calculate the ratio of spontaneous emission to stimulated emission for tungsten filament lamp operating at a temperature of 1500 K (assume the average frequency* ν *to be* $5\text{x}10^{14}$ *Hz).*

Solution: *The ratio R of spontaneous emission to stimulated emission (* $R = \frac{A_{21}}{\rho_\nu B_{21}}$ *) is as follows*

$$R = \exp(h\nu / kT) - 1$$

Taking $k=1.38\text{x}10^{-23}$ JK^{-1}, $h=6.6\text{x}10^{-34}$ *J.s,* $\nu = 5\text{x}10^{14}$ *Hz.*

$$R = \exp\left(\frac{6.6\text{x}10^{-34}.5\text{x}10^{14}}{1.38\text{x}10^{-23}.1500}\right) - 1$$
$$\approx e^{16} \approx 8.4\text{x}10^{6}$$

This confirms that under conditions of thermal equilibrium stimulated emission is not very significant. For sources operating at lower temperatures and higher frequencies stimulated emission is even less likely.

Example 4.3*: At what temperature are the rates of spontaneous and stimulated emission equal for* λ*=550 nm radiation? At what wavelength are they equal at room temperature (T=300 K)?*

Solution*: For spontaneous and stimulated emission rates to be equal at wavelength of 550 nm, we have*

$$R = \exp\left(\frac{h.c}{\lambda kT}\right) - 1 = 1$$
$$\Rightarrow T = \frac{h.c}{\lambda k}\frac{1}{\ln(1+R)}$$

Substituting values of $h=6.6\text{x}10^{-34}$ J.s, $k=1.38\text{x}10^{-23}$ JK^{-1}, $c=3\text{x}10^{8}$ m/s, λ=550 nm, and R=1, we get T=37635 K. Hence to achieve the equal spontaneous and stimulated emission temperature of more that thirty seven thousand Kelvin is required.

Similarly for equal spontaneous and stimulated emission rates at room temperature we can find the required wavelength, λ, as

$$\lambda = \frac{h.c}{k.T}\frac{1}{\ln(1+R)}$$

Again substituting values of all the constants and temperature T=300 K, we found λ~69 μm which lies in the microwave region. Thus radiation of 69 μm wavelength have equal spontaneous and stimulated emission rates at room temperature.

The above discussion indicates that the process of stimulated emission competes with the processes of spontaneous emission and absorption. Clearly if we wish to amplify a beam of light by stimulated emission then we must increase the rate of this process in relation to the other two processes. To achieve this for a given pair of energy levels we must increase i) radiation density and ii) the population density N_2 of the upper level in relation to the population density N_1 of the lower level. We shall show that to produce laser action we must create a condition in which $N_2>N_1$, even though $E_2>E_1$ that is we must create a so-called population inversion.

4.2.2 Laser Pumping

The population inversion required for light amplification constitutes an abnormal distribution of atoms among the various available energy levels. To understand how light amplification can be achieved in a medium, it is necessary to consider the Boltzmann distribution and then pumping mechanism to achieve the population inversion.

4.2.2.1 Population Inversion

The population inversion condition required for light amplification is a non-equilibrium distribution of atoms among the various energy levels of the atomic system. The Boltzmann distribution, which applies to a system in thermal equilibrium, is given by equation 7, it is obvious that as E_j increases N_j decreases for a constant temperature. We note that if the energy difference between E_1 and E_2 is nearly equal to kT (~0.025 eV at room temperature) then the population of the upper level would be 1/e or 0.37 times of the lower level. For an energy difference large enough to give visible radiation (~2.0 eV), however, the population of the upper level is almost negligible.

Example 4*: An atom has two energy levels with a transition wavelength of 694.3 nm. Assuming that all of the atoms in an assembly are in one or other of these levels, calculate the percentage of the atoms in the upper level at room temperature (T=300 K) and at T= 500 K.*

Solution: *The energy of the radiation of wavelength 694.3 nm,*

i.e. $E=hc/\lambda=6.6\text{x}10^{-34}.3\text{x}10^{8}/694.3\text{x}10^{-9}=2.85\text{x}10^{-19}$ *Joules*

We have

$$\frac{N_2}{N_1}=\exp\left(\frac{-E}{kT}\right)=\exp\left(\frac{-2.85\text{x}10^{-19}}{1.38\text{x}10^{-23}.T}\right)$$

At room temperature i.e. T=300 K, $N_2/N_1=1.2\text{x}10^{-30}$*, i.e. population of the upper level is* $1.2\text{x}10^{-28}$*% of the lower level in thermal equilibrium.*

At 500 K, $N_2/N_1=1.12\text{x}10^{-17}$*, i.e., population of the upper level is* $1.12\text{x}10^{-15}$*% of the lower level in thermal equilibrium.*

This shows that population of the upper level increases about 10^{13} *times by increasing the temperature up to 500 K from room temperature, but in both cases* $N_2 << N_1$*.*

Example 4.5: *The relative number of atoms* N_1 *and* N_2 *in two energy levels* E_1 *and* E_2 *separated by an energy gap* E_2*-*E_1 *are given at thermal equilibrium by Boltzmann ratio. Evaluate the ratio* N_2/N_1 *for the following cases:*

an optical transition, λ*=550 nm, at room temperature, 300 K;*

a microwave transition, ■ = 3 GHz, at room temperature;

A 10 GHz transition at liquid-helium temperature, 4.2 K.

For an optical transition at λ *=550 nm to have* N_2/N_1 *= 0.1, what temperature would be* *required?*

Solution:

The energy difference for λ = 550 nm is E_2-E_1 = *hc/550*x*10*$^{-9}$ = *3.6*x*10*$^{-19}$ *J*

$$\frac{N_2}{N_1} = \exp\left(\frac{-3.6\text{x}10^{-19}}{1.38\text{x}10^{-23} \times 300}\right)$$
$$\approx e^{-87} \approx 10^{-37}$$

The number of atoms present in the upper energy level for a visible transition at room temperature is negligibly small.

The energy difference for ■ = 3 GHz, $h\nu$ = *6.6*x*10*$^{-34}$*.3*x*10*^{9} = *1.98*x*10*$^{-24}$ *J*

$$\frac{N_2}{N_1} = \exp\left(\frac{-1.98\text{x}10^{-24}}{1.38\text{x}10^{-23} \times 300}\right)$$
$$\approx 0.99952$$

In the microwave region, number of atoms present in the upper energy level is a little less than the lower energy level at room temperature. Therefore to create a population inversion in microwave region is easier than in optical region.

The energy difference for ■ = 10 GHz, $h\nu$ = *6.6*x*10*$^{-24}$ *J*

$$\frac{N_2}{N_1} = \exp\left(\frac{-6.6\text{x}10^{-24}}{1.38\text{x}10^{-23} \times 4.2}\right)$$
$$\approx 0.89237$$

At low temperature the number of atoms present in the upper energy level decreases.

To have N_2/N_1 *= 0.1 at* λ *= 550 nm, the temperature required is 11329 K.*

This shows that to increase the number of atoms in the upper energy level at thermal equilibrium requires very high temperatures.

From the above example it is obvious that population can not be achieved by increasing the temperature of the medium. To create a population inversion in the optical region, we must supply energy selectively to the lower energy level, to excite atoms into the upper energy level. This excitation process is called pumping. Pumping produces a non-thermal equilibrium situation.

One of the methods used for pumping is stimulated absorption, that is the energy levels which one hopes to use for laser action are pumped by intense irradiation of the system. Now as B_{12} and B_{21} are equal once atoms are excited into upper level the probabilities of further stimulated absorption or emission are equal so that even with very intense pumping we can not get more atoms in the excited state. The best we can achieved is the equal population in both the levels, therefore, to make a laser amplifier with just two energy levels is not possible, i.e. a population inversion in a two level system can never be achieved by optical pumping.

Therefore, we must look for materials with either three or four energy levels system. This is not a problem as atomic systems generally have a large number of energy levels. A three level system is illustrated in Figure 4.6. Initially the distribution obeys the Boltzmann law. If the collection of atoms is intensely illuminated by photons, they can be excited into the level E_3 from the ground level E_1. From the level E_3, the atoms decay by non-radiative process to the level E_2 and a population inversion may be created between E_2 and E_1. Ideally the transition from level E_3 to E_2 should be very rapid to keep E_3 level almost empty. The transition from E_2 to E_1 should be slow, that is E_2 should have relatively longer lifetime. This allows a large build-up in the number of atoms in level E_2. Hence N_2 may become greater than N_1 and then population inversion will be achieved.

The level E_3 should preferably consist of a large number of closely spaced levels so that pumping uses a wide range of the radiation. This increases the pumping efficiency. Three level lasers, for example ruby, require very high pumping powers because the terminal level of the laser transition is the ground state. This means that more than half of the ground state atoms has to be pumped to the upper state to achieve population inversion.

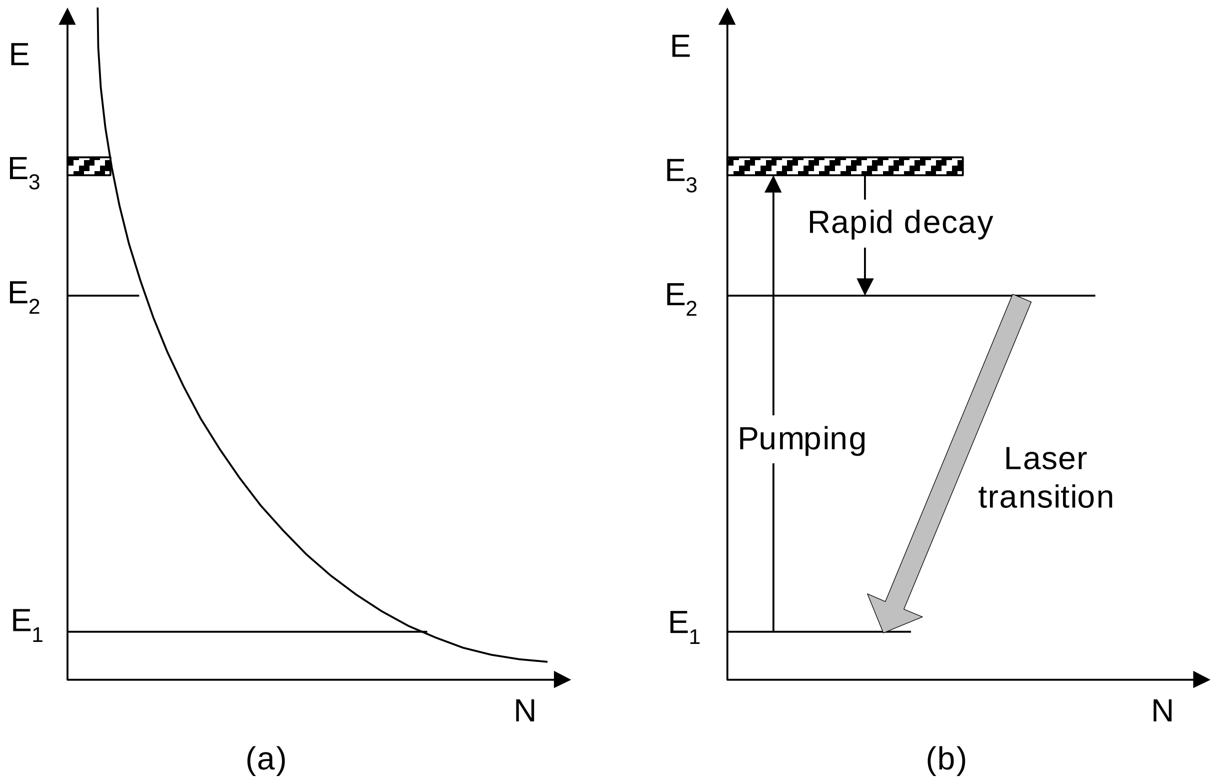


Figure 4.6 Population of energy levels by pumping in a three level system: (a) Boltzmann distribution before pumping and (b) distribution after pumping and the transitions involved.

The four-level system shown in Figure 4.7 has much lower pumping requirements. The populations of the levels E_4, E_3, and E_2 are all very close in conditions of thermal equilibrium. Thus, if atoms are pumped from the ground state to the level E_4 from which they decay very rapidly to the level E_3. The population inversion is quickly created between levels E_3 and E_2. The energy level schemes of the media used in lasers are often complex but they can usually be approximated by either three or four level schemes.

4.2.3 Optical Resonator

The population inversion is not all it takes to make a laser. A material with an inverted population can emit light in every direction. The light may be due to stimulated emission, and it may be at a single wavelength, but is not converted into a laser beam. To extract energy efficiently from a medium with a population inversion and make a laser beam, we need a resonant cavity that helps building (or amplifying) stimulated emission by feedback, i.e., reflecting some of the light back into the laser medium.

Before discussing the feedback, let us start by looking at the process of amplification.

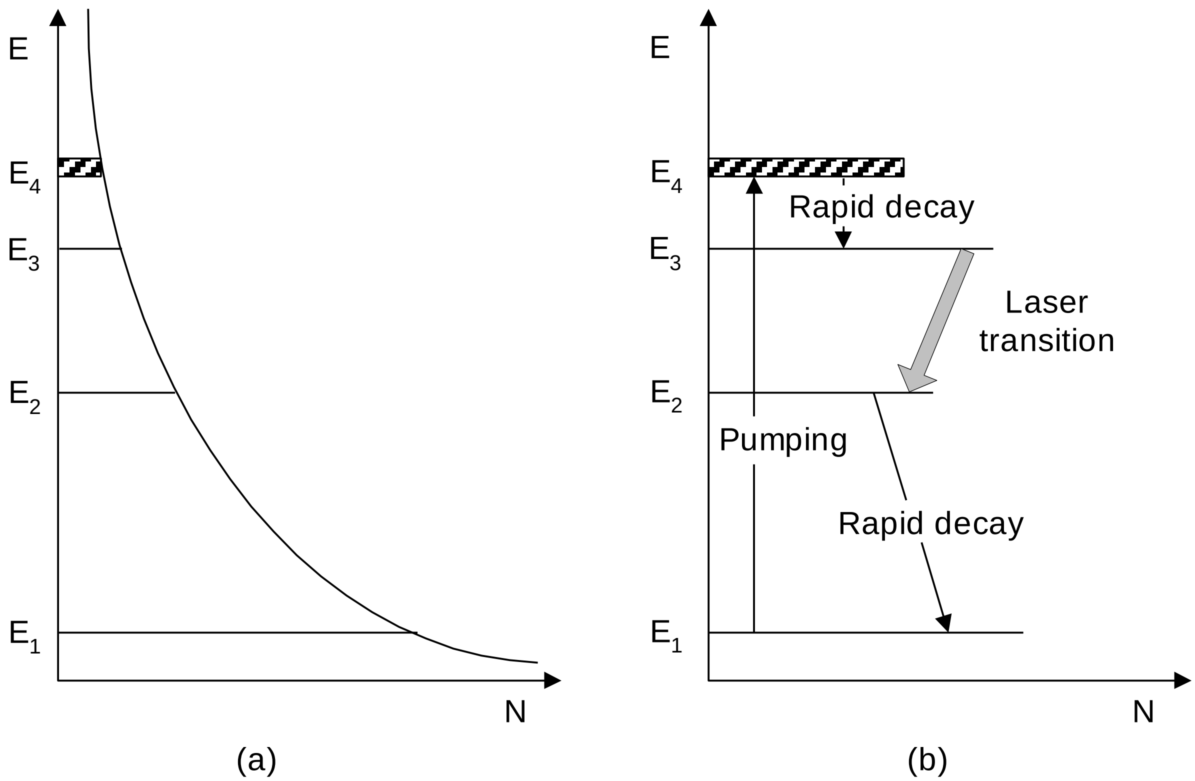


Figure 4.7 Population of the energy levels in a four level system: (a) before pumping and (b) after pumping.

4.2.3.1 Amplification of Light

In lasers, light is amplified. To understand it let us consider a collimated beam of perfectly monochromatic radiation passing through an absorbing medium (Figure 4.8). We assume for simplicity that there is only one relevant electron transition, which occurs between the energy levels E_1 and E_2. For homogeneous medium, the change in irradiance of the beam dI is proportional to the distance travelled dx and to the incident intensity I, i.e. $dI = -\alpha I.dx$. Here the constant of proportionality, α, is the absorption coefficient. The negative sign indicates the reduction in beam irradiance due to absorption. Writing this equation in differential form we have

$$\frac{dI}{dx} = -\alpha I \qquad (4.13)$$

By integrating Eq. 4.10 we have

$$I = I_o e^{-\alpha x} \qquad (4.14)$$

where I_o is the incident intensity.

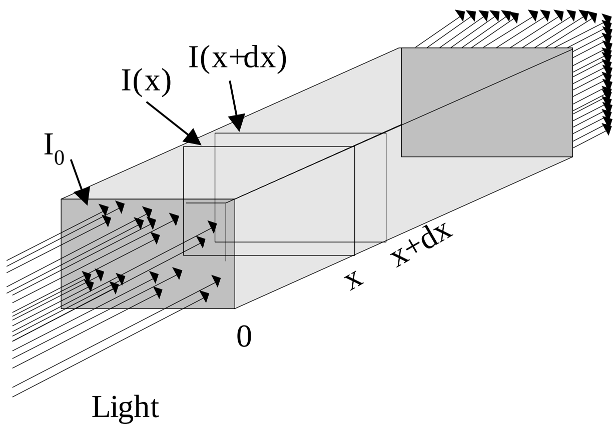


Figure 4.8 Collimated beam of light traversing an absorbing medium. The change in the irradiance across a small slab of the medium is proportional to the irradiance at the slab and to the thickness of the slab dx.

Example 4.6: *If 1% of the light incident into a medium is absorbed per millimetre, what fraction is transmitted if the medium is 0.1 m long?*

Solution*: Transmission of light passing through an absorbing medium of length x is given by*

$$I = I_0 \, e^{-\alpha x}$$

For $\alpha = 0.01\ mm^{-1}$, *x=0.1 m=100 mm, we have*

$$I = I_0 \, e^{-0.01x100} \quad = I_0 . e^{-1} = 0.368 \mathrm{x} I_0$$

Therefore, 36.8% of the incident light will transmit through the medium and rest will be absorbed.

Let us consider the absorption coefficient in more detail. Clearly the degree of absorption of the beam will depend upon N_1, number of atoms with electrons in the lower energy level E_1, and N_2, number of atoms with electrons in upper energy level E_2. If N_2 is zero then the absorption would be maximum, while if all of the atoms are in the upper level the absorption would be zero and the probability of stimulated emission would be large.

When a beam having N number of photons per unit volume pass through a medium then net rate of loss of photons can be determined from the stimulated transitions, i.e. absorption and emission.

$$\frac{dN}{dt} = N_2\rho_\nu B_{21} - N_1\rho_\nu B_{12} \qquad (4.15)$$

For the same degeneracy of both levels (i.e. $B_{12}=B_{21}$), we can write

$$\frac{dN}{dt} = (N_2 - N_1)\rho_\nu B_{21} \qquad (4.16)$$

In this discussion we have ignored photons generated by spontaneous emission as these are emitted randomly in all direction and do not contribute to the initial collimated beam. Similarly we have ignored scattering losses.

Irradiance of a beam is the energy crossing a unit area in unit time. In mathematical form it can be written as

$$I=N.h\nu_{21}.u \qquad (4.17)$$

where u is the velocity of light in the medium. If n is the refractive index of the medium then u=c/n, where c is the velocity of light in free space. The rate of change of irradiance when passing through a medium is given by

$$\frac{dI}{dx} = \frac{dN}{dx} h\nu_{21}.u \qquad (4.18)$$

since

$\frac{u}{dx} = \frac{1}{dx/u} = \frac{1}{dt}$, we can write equation 4.18 as

$$\frac{dI}{dx} = \frac{dN}{dt} h\nu_{21} \qquad (4.19)$$

Combining equations 4.16 and 4.19 we get

$$\frac{dI}{dx} = (N_2 - N_1)\rho_\nu.B_{21}.h\nu_{21} \qquad (4.20)$$

Now from equations 4.13, 4.20, ρ_ν = Nhν, and equation (4.17)

$$(N_1 - N_2).\rho_\nu.B_{21} = \alpha.\rho_\nu.u\frac{1}{h\nu_{21}} \qquad (4.21)$$

Re-arranging for absorption coefficient α, and substituting the value of u = c/n, we have

$$\alpha = (N_1 - N_2)\frac{B_{21}.h\nu_{21}.n}{c} \qquad (4.22)$$

The absorption coefficient, α, depends on the difference in the populations of the two energy levels E_1 and E_2. For a collection of atoms in thermal equilibrium, N_1 will always be greater than N_2. If, however, we can create a situation in which N_2 becomes greater than N_1 then α is negative and the quantity (-αx) in the exponent of equation 11 becomes positive. Thus the irradiance of the beam grows as it propagates through the medium in accordance with the equation:

$$I = I_o e^{kx} \qquad (4.23)$$

where k is referred to as the small signal gain coefficient and is given by

$$k = (N_2 - N_1)\frac{B_{21}.h\nu_{21}.n}{c} \qquad (4.24)$$

In this situation the medium acts as an amplifier and may be called active medium.

Example 4.7*: If the irradiance of light doubles after passing once through a laser amplifier 0.5 m long, calculate the small signal gain coefficient assuming no losses in the medium. If the increase in irradiance were only 5% what would be k?*

Solution: In an optical amplifier, the light intensity increases as

$$I = I_0 e^{kx} \Rightarrow k = \frac{1}{x}\ln\left(\frac{I}{I_0}\right)$$

(a) for I/I_0=2 and x=0.5 meter; *k=1.386 m^{-1}*

The value of signal gain coefficient is 1.386/m

(b) for I/I_0=1.05 and x=0.5 meter; *k=0.098 m^{-1}*

The value of signal gain coefficient is 0.098/m.

4.2.3.2 Optical Feedback

The gain (amplification) per unit length of most active media is so small that very little amplification of a beam of light results from a single pass. In the multiple passes through the same active media may result a large amplification.

To make an oscillator from an amplifier, it is necessary to introduce a suitable positive feedback. In the laser, the positive feedback may be obtained by placing the active material between two highly reflecting mirrors (e.g., plain-parallel mirrors as shown in Figure 4.9). The initial stimulus is provided by any spontaneous transitions between appropriate energy levels in which the emitted photon travels along the axis of the system. The signal is amplified as it passes through the medium and fedback by mirrors. Saturation is reached when the gain provided by the medium exactly matches the losses incurred during a complete round trip.

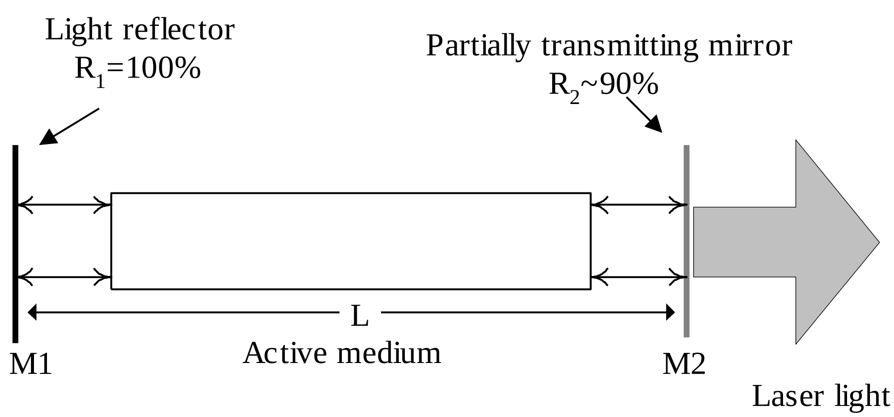


Figure 4.9. Scheme of laser oscillations.

If one of the two mirrors is made partially transparent, a useful output beam can be extracted. It is important to note that for laser, a certain threshold condition must be fulfilled. For laser, the oscillation will start when the gain of the active medium compensates the losses. Thus, while a population inversion is a necessary condition for laser action it is not the sufficient one because the minimum or threshold value of gain coefficient must be large enough to overcome the losses and sustain oscillations. The threshold gain specifies the minimum population inversion required.

The total loss of the system is due to a number of different processes, the most important ones include:

transmission at the mirrors, in fact transmission from one of the mirrors usually provides the useful output, the other mirror is made as reflective as possible to minimise losses;

absorption and scattering at mirrors;

absorption in the laser medium due to transmissions other than the desired transitions;

scattering at optical inhomogeneities in the laser medium;

diffraction losses at the mirrors.

To simplify, let us include all the losses except those due to transmission at the mirrors in a single effective loss coefficient γ, which reduces the effective gain coefficient to (k-γ). We can determine the threshold gain by considering the change in irradiance of a beam of light undergoing a round trip within the laser cavity. We assume that the laser medium fills the space between the two mirrors M_1 and M_2 which have reflectance R_1 and R_2 and a separation L. In travelling from mirror M_1 to M_2 the beam irradiance increases from I_0 to I according to equation 4.23 as,

$$I = I_0 .e^{(k-\gamma).L} \qquad (4.25)$$

After reflection at M_2, the beam irradiance will be $R_2.I_0.\exp[(k-\gamma).L]$ and after a complete round trip the final irradiance will be such that the round trip gain G is

$$G = R_1.R_2.e^{2(k-\gamma)L} \qquad (4.26)$$

If G is greater than unity then there will be a net amplification and the oscillation will grow; if G is less than unity, the oscillation will die out. Therefore, we can write the threshold condition as

$$G = R_1.R_2.e^{2(k_{th}-\gamma)L} = 1 \qquad (4.27)$$

where k_{th} is the threshold gain. It is important to realise that the threshold gain is equal to the steady-state gain in continuous output lasers. This equality is due to gain saturation, which can be explained as follows. Initially, when laser action starts the gain may be well above the threshold value. The effect of stimulated emission reduces the population of the upper level of the laser transition so that the degree of population inversion and consequently the gain will decrease. Thus the round trip gain may vary. It is only when G has been equal to unity for a period of time that the laser output power settles down to a steady-state value, that is when the gain just balances the losses in the medium. In terms of the population inversion there will be a threshold value

$$N_{th} = (N_2 - N_1)_{th} \tag{4.28}$$

In the steady state situation (N_2-N_1) remains equal to N_{th} regardless of the amount by which the threshold-pumping rate is exceeded. The small signal gain required to support steady state operation depends on the laser medium through k and γ, and on the laser construction through R_1, R_2 and L.

$$k_{th} = \gamma + \frac{1}{2L}\ln\left(\frac{1}{R_1.R_2}\right) \tag{4.29}$$

The above equation shows that k can have a wide range of values, depending not only (N_2-N_1) but also on the intrinsic properties and design parameters. If k is high then it is relatively easy to achieve laser action. With low gain media, mirrors must have high reflectivities, be very clean and carefully aligned.

Example 8: *Calculate the mirror reflectance required to sustain laser oscillations in a laser which is 0.1 m long given that the small signal gain coefficient is 1* m^{-1} *(assume one mirror has 100% reflectance).*

Solution: *Gain of a laser amplifier is given by* $G=R_1R_2e^{2kL}$ *and for threshold it should be unity, therefore,*

$$R_1R_2\, e^{-2k_{th}L} = 1$$

or

$R_1R_2= e^{-2k_{th}L}$

It is given that $k_{th}=1\ m^{-1}$, $R_1=1$, *and* $L=0.1$ *m. The value of* R_2 *is:*

$R_2=e^{-0.2}=0.82$

The reflectance of the second mirror (i.e. output coupler) is 82 %.

The efficiency of laser system is the ratio of the output light power to input pump power. It therefore depends on how effectively the pump power is converted into producing a population inversion.

Problems

4.1 The part of the electromagnetic spectrum that is of interest in the laser field starts from the submillimeter wave region and goes down in wavelength to the x-ray region. This covers the following regions in succession: (1) far infrared; (2) near infrared; (3) visible; (4) ultraviolet (uv); (5) vacuum ultraviolet (vuv); (6) soft x-ray; (7) x-ray. Draw a chart (to the scale) indicating all the regions in the units of (i) Angstroms, (ii) Hertz, (iii) electron volts (ev), and (iv) meters.

4.2 When in thermal equilibrium (at T=300 K), the ratio of the level populations N_2/N_1 for some particular pair of levels is given by 1/e. Calculate the frequency ν for this transition. In what region of the e.m. spectrum does this frequency fall?

4.3 The value of signal gain coefficient of a certain laser amplifier is 0.29/m. What's physical meanings of it?

4.4 Calculate the small signal gain co-efficient required to sustain laser oscillations in a medium which is 0.15 meter long, given that the effective loss coefficient is 0.15 m^{-1}, the reflectivity of rear mirror is 100% and that of output mirror is 80%.

4.5 For a two level system, suppose $N_2 = 10N_1$, where N_1 and N_2 denote the number of atoms (per unit volume) in level 1 and level 2 respectively. The transition between these two levels corresponds to the frequency of $5x10^4$ Hz. Calculate the ratio of spontaneous emission and stimulated emission. Also calculate the temperature required to produce this population inversion.

4.6 In a two levels system, the levels are separated by an energy E_2 - E_1. Find the ratio of population inversions at room temperature if transition between levels occur at (i) λ = 0.55 μm (ii) λ = 3 μm

4.7 When two quantum energy levels, E_1 and E_2 of an atom are separated by an energy gap $\Delta E = E_2 - E_1$, and a large number of such atoms are in thermal equilibrium at temperature T, then the relative number of atoms N_1 and N_2 in the two energy levels are given by the Boltzmann ratio $N_2/N_1 = e^{-\Delta E/kT}$. Evaluate this ratio for the following cases:

(a) transition occurs at the frequency of 3100 MHz, and the temperature is 300 K. What is the fractional population difference $(N_2-N_1)/ N_1$?

(b) Consider the same situation, except that ν = 9000 MHz and the temperature is 4 K. What is $(N_2-N_1)/ N_1$?

(c) Calculate the Boltzmann ratio N_2/ N_1 for λ = 5500 A^o, and T = 300 K.

(d) What temperature required to make (N_2 equal to 8 % of N_1 in part (c) ?

4.8 The irradiance of light becomes half after passing through a laser media. Calculate the small gain coefficient, explain for obtaining negative answer in part.

4.9 If the irradiance of light becomes double after passing once through a laser amplifier 0.5 m long., assuming no losses in the media:

(a) Calculate the small signal gain co-efficient.

(b) Calculate the small signal gain co-efficient, if the length of the media is doubled.

(c) By doubling the length of the media, is the small signal gain co-efficient doubled or not? Give comprehensive comments in either case.

Books for further reading

J. Wilson and J.F.B. Hawkes, *Optoelectronics: An Introduction*, (Prentice-Hall International, London, 1983)

O. Svelto, *Principles of Lasers*, (Plenum Press, New York, 1989)

P. W. Milonni and J. H. Eberly, *Lasers*, (John Wiley & Sons, New York, 1991).

A. E. Seigman, *Lasers and Masers*, (1971).

D. C. O'Shea, W. R. Callen, and W. T. Rhodes, *Introduction to Lasers and Their Applications*, (Addison-wesley, London, 1978)

Unit-5

Pumping Processes

Objective

The pumping processes are one of the essentials of a laser system. In this unit a couple of these processes are discussed. Attainment of population inversion through rate equation treatment has been described for three and four-level systems. Gain saturation behavior is also discussed. The student is required to understand the rate equation analysis, such that pumping rate and gain of a system can be calculated.

5.1 Introduction

The process by which atoms are raised from lower energy level to higher energy level is called pumping process. The most common pumping schemes are: optical or electrical. In Optical pumping the light from a powerful source is absorbed by the active material and the atoms are thereby pumped into the upper energy level. This method is particularly suited to solid-state (e.g., ruby or neodymium) and liquid dye lasers. Energy bands in solids and in liquids absorb a sizeable fraction of the broadband light emitted by the flash lamp.

Electrical pumping is particularly suited to gas and semiconductor lasers. In gas lasers, sufficiently intense electrical discharge is responsible for pumping. In gases, optical pumping is not preferred because of small widths of absorption line and usually broadband emission of the flash lamp. In the case of semiconductor lasers, optical pumping could be used but electrical pumping is very convenient, since it is very easy to pass a current through the semiconductor.

5.2 Pumping Mechanisms

There are quite a number of ways for pumping the active medium but most common are the optical and electrical. Following is the list of some common pumping processes; however, only the optical and electrical pumping with a little more details will be discussed.

5.2.1 Chemical Pumping

Energy from an exothermic chemical reaction in a gas can excite a laser medium. In the simplest way, two species react to produce a third species, which carries at least part of the exothermic energy of reaction as vibrational energy. If the reaction is rapid, the result can be a population inversion of the excited species produced by the reaction. The most common chemical lasers are the family of hydrogen halides. The chemical reaction can be shown as

$$A + B \rightarrow (AB)^*$$

where (AB)* is the resultant molecule in the excited vibrational state.

5.2.2 Gas Dynamic Pumping

Another way to produce a population inversion is to expand a hot, high-pressure gas into a vacuum. The gas dynamics expansion cools the gas, but this cooling is not same for all the energy levels. At high temperature sufficient number of molecules can reach to the higher energy level, if enough of the molecules in this energy state remain there even after expansion of the gas and the molecules in the lower energy state relax immediately after expansion, a population inversion can be achieved. The inversion lasts only until the gas can start approaching equilibrium. Within a gas dynamic laser, the population inversion exists only for a short distance downstream of the expansion nozzle. In practice, only one type of carbon dioxide laser, called gas dynamic CO_2 laser, is pumped with this technique, and produces powers of tens or hundreds of kilowatts.

5.2.3 Nuclear Pumping

Laboratory demonstrations have shown that gas laser can be excited by transferring energy from the products of nuclear reaction to the excited species. In experimental devices, ions produce by the fission reactions then transfers their energy to the laser gas, producing population inversion. Research on the concept revived in the mid-1980s by the Strategic Defense Initiative, but little progress has been made to the practical devices.

Now we will discuss the two popular pumping schemes in some details.

5.2.4 Optical Pumping

Optical pumping can be subdivided into two branches, one is laser pumping and the other is flashlamp pumping. The former is less common as compare to the latter.

5.2.4.1 Laser Pumping

It is a special kind of optical pumping where a laser beam is used to pump another laser. The directional properties of a laser beam make it very convenient for pumping another laser. The monochromaticity of the pump laser means that laser pumping is not limited to solid-state and liquid lasers but can also be applied to gas lasers. In the latter case, the line emitted by the pumping laser must coincide with an absorption line of the laser to be pumped. This situation applies in most far infrared gas lasers, e.g. methyl alcohol and CH_3OH lasers, which are usually pumped by a suitable line of a CO_2 laser.

In laser pumping the wavelength of the pumped laser is higher than the pumping laser (i.e., $\lambda_{emission} > \lambda_{absorbed}$) but the pumping efficiency is significantly higher as compare to the other scheme.

5.2.4.2 Flashlamp Pumping

In the case of optical pumping, the light from a powerful incoherent lamp is focussed, by a suitable optical system, to the active material. Figure 5.1 shows three most commonly used pumping configurations. In all three cases the active material is taken to be the form of a cylindrical rod, as usually applied in the practical systems. Its diameter ranges from a few millimeters to a few centimeters and lengths from a few millimeters up to a few tens of centimeters. This can be operated in a pulsed or continuous wave regime depending on whether the lamp is pulsed or continuous. In Figure 5.1a the lamp has a helical shape, and the

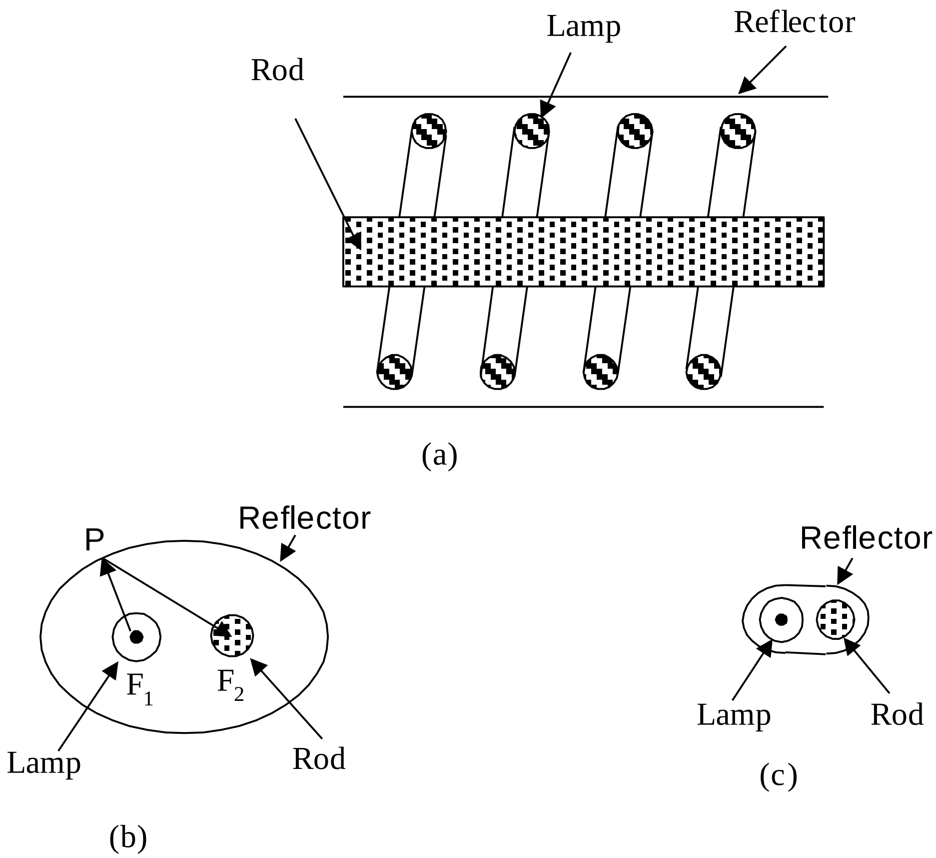


Figure 5.1. Most commonly used optical pumping systems. (a) The lamp is helical shape, light reaches to the rod directly or after reflection from cylinderical surface. (b) The lamp is a linear cylinderical shape, it is placed at focus of the elliptical reflecting cyilider and rod is at the other focus. (c) The and linear lamp are placed as closed as possible and are surrounded by a clo coupled cylinderical reflector.

light reaches the active material either directly or after reflection at the specular cylindrical surface. This was the arrangement used for ruby laser, and it is still in use for some type of pulsed lasers. In Figure 5.1b the lamp is in the form of a cylinder (linear lamp) having radius and length roughly equal to those of the active rod. The lamp is placed along one of the two focal axes, F_1, of a specularly reflecting elliptical cylinder and the laser rod is placed along the second focal axis F_2. A well-known property of an ellipse is that all the rays emitting from one focus converge at the second focus after reflection from the surface of the elliptical cylinder. Therefore, a large fraction of the light emitted by the lamp illuminate the active material. Figure 5.1c shows an example of what is known as a close-coupled configuration. The rod and the linear lamp are placed as closed as possible and are surrounded by a close-coupled cylindrical reflector. The efficiency for close-coupled configurations is usually not much lower than for the elliptical cylinders.

Pumping Efficiency

For continuous wave lasers, pumping efficiency η_p can be defined as the ratio between the minimum power P_m that would be needed to produce a given pumping rate and the electrical pump power P actually delivered to the lamp. The minimum power can be written as

$$P_m = \left(\frac{dN_2}{dt}\right).V.h\nu_p = W_p N_1.V.h\nu_p \tag{5.1}$$

where W_p is the pumping rate, V is the volume of the active material and ν_p is the frequency difference between the ground level and the upper laser level. Therefore, the efficiency can be written as

$$\eta_p = \frac{W_p.N_1.V.h\nu_p}{P}\,. \tag{5.2}$$

The pumping efficiency η_p can be written as the product of four terms corresponding to: (1) the emission of radiation from the lamp, (2) the transfer of this radiation to the active rod, (3) the absorption in the rod, and (4) the transfer of the absorbed energy to the upper laser level. Mathematically,

$$\eta_p = \eta_r.\eta_t.\eta_a.\eta_{pq} \tag{5.3}$$

where η_r is the lamp radiative efficiency, i.e., the efficiency of conversion from electrical input to light output in the wavelength range corresponding to the pump bands of the laser medium. η_t is transfer efficiency, which can be defined as the ratio of the pump power actually entering the rod to that emitted by the lamp in the useful pump range. η_a is the absorption efficiency, i.e., the fraction of light entering the rod that is actually absorbed by the material. η_{pq} is the power quantum efficiency, i.e., the fraction of the absorbed power that leads to population of the upper laser level. Typical values of the pumping efficiency for Nd:YAG laser ranges from 0.1 to 1 percent.

5.2.5 Electrical Pumping

Electrical pumping is used for gas and semiconductor lasers. In this section we will limit our discussion for gas lasers only, pumping for semiconductor lasers may be discussed in the description of diode lasers.

Electrical pumping of gas lasers is achieved by allowing a current to pass through the gas mixture. For this purpose, a high dc voltage is applied to break down the gases so it will conduct the electricity. Generally, there are two types of electrical pumping (i) longitudinal discharge pumping (ii) transverse discharge pumping as shown in Figure 5.2. In longitudinal discharge lasers, the electrodes often have an annular structure with the cathode surface usually much larger than that of the anode, which helps in reducing the degradation due to ion collisions. In a transverse discharge, the electrodes extend over the whole length of the laser material. Various electrode structures are used depending upon the types of laser involved. Usually longitudinal discharge arrangements are used for continuous wave (cw) lasers, while transverse discharges are used for both types of lasers i.e., cw and pulsed. The longitudinal discharge, when confined in a glass tube (or any other dielectric tube) provides a more uniform and stable pumping configuration.

In an electrical discharge, ions and free electrons are produced. From the applied electric field, they acquire kinetic energy and are able to excite a neutral atom or molecule by collision. The positive ion, due to their heavier mass, are accelerated to lower velocities than the electrons and therefore do not play any significant role in the excitation process. Electrical pumping of a gas occurs via one, or both, of the following processes:

(i) For a gas consisting of only one species, the excitation is only produced by electron impact, i.e., according to the process

$$e + x \rightarrow x^* + e$$

where x and x* represent the atom in the ground and excited states, respectively.

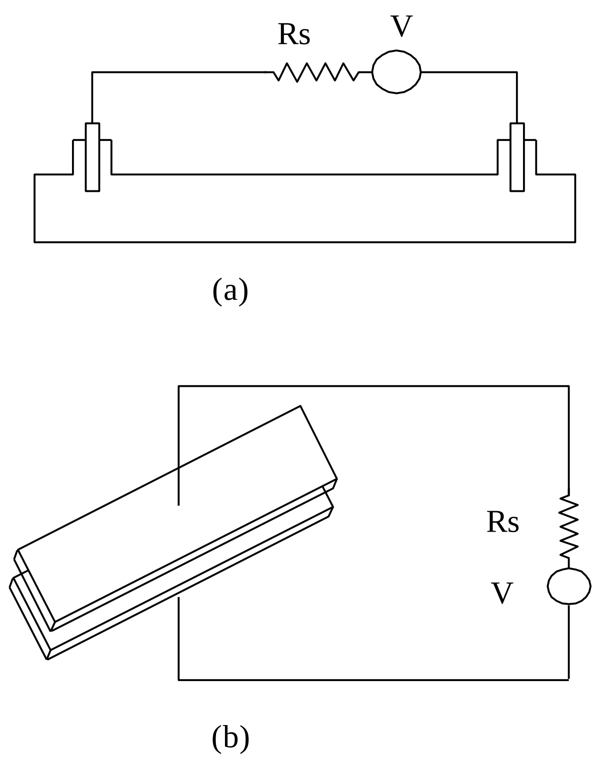


Figure 5.2 Frequently used pumping configuration for gas-discharge excitation; longitudinal discharge, and (b) transverse discharge.

(ii) For a gas consisting of two species (say A and B), excitation can also occur as a result of collisions between atoms of different species through a process known as resonant energy transfer as shown in Figure 5.3. Let us assume that species B is in ground state and species A is in the excited state brought by the electron impact. It is also assume that the energy difference ΔE between the two transitions is less than kT (which is the thermal energy at room temperature). In this case, there is an appreciable probability that, after collision, species A will be found in its ground state and species B in its excited state. This process can be denoted by

$$A^{*}+B \rightarrow A+B^{*} \pm \Delta E \qquad (5.4)$$

where the energy difference ΔE will be added or subtracted from the translational energy, depending on its sign. This process provides a particularly attractive way of pumping species B, if the upper state of A is metastable (forbidden transition). In this case, once A is excited to its upper level, it will remain there for a long time, thus constituting an energy reservoir for excitation of the B species.

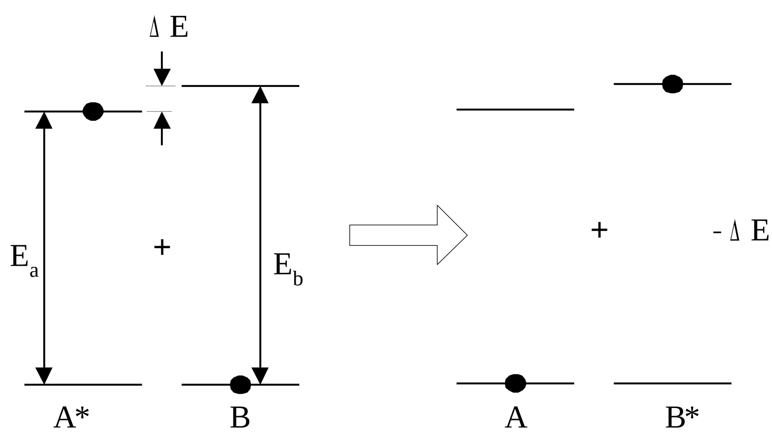


Figure 5.3 Near resonant energy transfer from A* to B

Helium-neon and carbon dioxide lasers are good examples of this kind of pumping process. In helium neon lasers, helium has a metastable state close to the upper laser level of neon and therefore, it plays the role of species A in the mixture. In carbon dioxide lasers, a mixture of CO_2, N_2 (nitrogen) and helium gases is used, here nitrogen molecule has a metastable level close to the upper laser level in the CO_2 molecule. Resonant energy transfer from N_2 to CO_2 increases the pumping efficiency.

5.2.5.1 Physical Characteristics of Discharges

In electrical discharge electrons play the main role. They acquire energy from the applied electric field and lose or exchange energy through three processes:

(1) Inelastic collisions with the atoms (or molecules) of the gas mixture, which either raise the atom to one of its excited states or ionize it. These electron-impact excitation or ionization phenomena are perhaps the most important processes for laser pumping.

(2) Elastic collisions with the atoms. If we assume these atoms to be at rest before the collision (the mean atomic velocity is much smaller than that of the electrons), the electron will lose energy upon collision. It can be shown by a straightforward analysis of the elastic collision that, if the direction of scattered electron is random, the

electron loses on average a fraction 2(m/M) of its energy, where m is the electron mass and M is the mass of the atom. This loss is very small since m/M is small, e.g., $m/M = 1.3x10^{-5}$ for Ar atoms.

(3) Electron-electron collisions. For less weakly ionized gas, the frequency of such collisions is usually high since both particles are charged and exert forces on one another at considerable distance. Moreover, since both colliding particles have the same mass, the energy exchange in the collision is considerable. As a result of the collision phenomena mentioned above, the electron "gas" in the plasma acquires a distribution of velocities and hence of energies.

5.3 Steady State Laser Pumping and Population Inversion

One of the most common applications for rate equations is the analysis of laser pumping. In this section, therefore, we will develop and solve the rate equations to analyze steady-state laser pumping in simplified four-level and three-level laser systems.

5.3.1 Elementary Four-Level Laser System

For the analysis of steady-state laser pumping and population, we consider a neodymium laser (Nd:YAG or Nd:Glass), which is a typical example of four-level laser system. The complicated energy levels of Nd^{+3} have been simplified into the idealized four-level laser system shown in Figure 5.4. This four-level model will in fact provide a simple but surprisingly accurate analytical model for many laser systems. In this model level 4 represents the combination of all the levels lying above the upper laser level in the real atomic system. It is desirable that many of these levels be in fact broad absorption bands, so that the optical pumping into these levels by a broadband pump lamp can be very efficient. Level 3 represents the upper laser level, usually a fairly sharp and long-lived level, with a large gap below it. Level 2 then represents the lower laser level and level 1 is the lowest or ground level. Other low-lying levels that may be present inside the material, both above and below the lower laser level, are ignored in the model because they play no role in the laser action. They act only as temporary way stations through which atoms may pass in relaxing from the other levels to the ground level E_1.

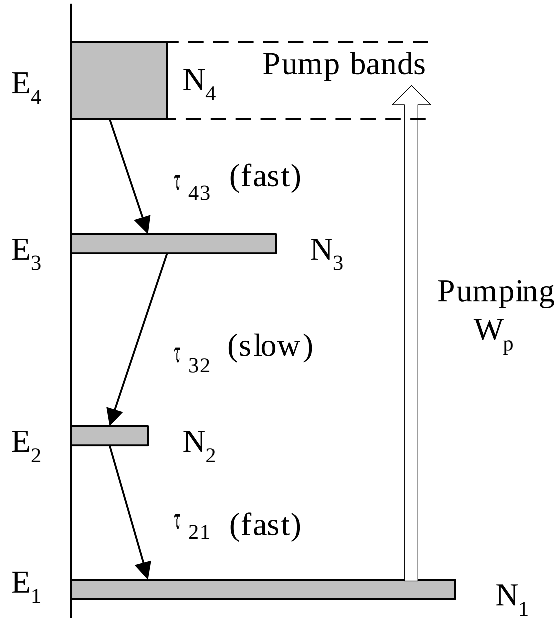


Figure 5.4 An idealized four-level pumping scheme suitabl for Nd:YAG laser system.

5.3.1.1 Four-Level Pumping Analysis

To analyze this system we will write down the relevant rate equations, using the model shown in Figure 5.4 and then solve for their steady-state solutions, making reasonable approximations during the analysis. Since the condition $(h\nu/kT) \gg 1$ is usually very well satisfied for all transitions in a visible laser system, we will write all of the following rate equations using the "optical frequency approximation". In this approximation, the thermally stimulated terms in the relaxation rates, either upward or downward, are totally negligible compared to the spontaneous emission rates, this is due to the fact that Boltzmann ratio at optical frequencies is always very small, i.e., on the order of 10^{-36} at room temperature.

We begin by assuming that the laser pumping process produces a stimulated transition probabilities $W_{14} = W_{41} = W_p$ between levels 1 and 4. The rate equation for level 4 in the optical approximation is then

$$\frac{dN_4}{dt} = W_p.(N_1 - N_4) - \frac{N_4}{\tau_4} \qquad (5.5)$$

where the lifetime τ_4 is give by

$$\frac{1}{\tau_4} = \frac{1}{\tau_{43}} + \frac{1}{\tau_{42}} + \frac{1}{\tau_{41}} \tag{5.6}$$

is the total lifetime for decay of level 4 to all lower levels. The steady-state population of level 4, when $dN_4/dt = 0$, is then given by

$$N_4 = \frac{W_p.\tau_4}{1 + W_p.\tau_4}.N_1 \tag{5.7}$$

The normalized pumping rate $W_p\tau_4$, which will appear in many of the following expressions, will in fact have a value much less than unity in many (though not all) practical laser systems.

Direct pumping up from the ground level into the upper laser level 3 in this model can very often be assumed negligible. This may be due to either of the following two reasons:

(1) The $1 \rightarrow 3$ transition have a weaker absorption cross section than the $1 \rightarrow 4$ transitions, where level 4 corresponds to a group of levels.

(2) The transition is much narrower than the strong absorption bands from the ground level to the groups of levels that make up level 4. The rate equations for the two levels N_3 and N_2 are then:

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3} \tag{5.8}$$

and

$$\frac{dN_2}{dt} = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} + \frac{N_4}{\tau_{21}} \tag{5.9}$$

Here, τ_3 is the total lifetime of the level 3, τ_{32} and τ_{21} are average time of transitions from level $3 \rightarrow 2$ and $2 \rightarrow 1$, respectively. Equation (5.8) then gives at steady state (d/dt = 0)

$$N_3 = \frac{\tau_3}{\tau_{43}}.N_4 \tag{5.10}$$

In a good laser system, the $4 \rightarrow 3$ relaxation rate will be very fast, but the upper laser level 3 will have a long lifetime by comparison, so that $\tau_3 >> \tau_{43}$ and hence $N_3 >> N_4$.

Combining equation (5.9) and (5.10) then gives the result

$$N_2 = \left(\frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43}.\tau_{21}}{\tau_{42}.\tau_3} \right).N_3 = \beta.N_3 \qquad (5.11)$$

where the parameter β is defined to be

$$\beta = \frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43}.\tau_{21}}{\tau_{42}.\tau_3} \qquad (5.12)$$

This parameter β thus depends only on relaxation-time ratios, not absolute values. If this quantity is less than unity, the steady-state result will be $N_2 < N_3$, which means there will be the desired population inversion on the $3 \rightarrow 2$ transition.

In a good laser system, the upper levels E_4 will relax primarily into the upper laser level E_3, so that $\tau_{42} \approx \infty$. In this case $\beta \approx \tau_{21}/\tau_{31}$, and the condition for population inversion becomes simply

$$\beta = \frac{N_2}{N_3} \approx \frac{\tau_{21}}{\tau_{32}} << 1 \qquad (5.13)$$

$$\Rightarrow N_2 << N_3 .$$

In other words, to have good inversion on the $3 \rightarrow 2$ transition, atoms should relax out of the lower laser level E_2 down into lower levels much faster than atoms relax into E_2 from above. Even if level 4 does not relax only into level 3, if the upper laser level has a long lifetime (both τ_{32} and τ_3 are long) and the lower laser level has a short lifetime (τ_{21} is short), then population inversion on the $3 \rightarrow 2$ transition is virtually certain.

Whether this population inversion will be large enough to give sufficient gain to achieve laser action in a practical cavity is another mater. Nonetheless, these conditions are met and laser action can be produced on many transitions in many real laser systems.

5.3.1.2 Fluorescence Quantum Efficiency

Another dimensionless parameter often used in evaluating laser materials is the fluorescent quantum efficiency η, defined as the number of fluorescent photons spontaneously emitted on the laser transition divided by the number of pump photons absorbed on the pump transitions when the laser material is below threshold. For the four-level system this quantum efficiency is given by

$$\eta = \frac{\tau_4}{\tau_{43}} \times \frac{\tau_3}{\tau_{rad}} \tag{5.14}$$

$\tau_{rad} = \tau_{rad}(3 \rightarrow 2)$ is the radiative life time for $3 \rightarrow 2$ transition. The first ratio in this expression tells what fraction of the total atoms excited to level 4 relax directly into the upper laser level 3, rather than by passing 3 and dropping to lower levels. The second ratio tells what fraction of the total decay out of level 3 is purely radiative decay to level 2.

5.3.1.3 Four-Level Population Inversion

Using the parameters β and η plus the conservation of atoms condition that is

$$N = N_1 + N_2 + N_3 + N_4 \tag{5.15}$$

We can solve for the population inversion N_3-N_2 for the four-level system by substituting values of N's from equation (5.7), (5.10), and (5.11) into equation (5.15) and taking the ratio with N_3-N_2, we get

$$\frac{N_3 - N_2}{N} = \frac{(1-\beta)\eta.W_p.\tau_{rad}}{1 + \left[1 + \beta + 2\tau_{43}/\tau_{rad}\right]\eta.W_p.\tau_{rad}} \tag{5.16}$$

where $\tau_{rad} = \tau_{rad}(3 \rightarrow 2)$ is the radiative decay time of the laser transition itself. In a good laser material the lifetime τ_{43} from the pump level into the upper laser level will be short compared to the radiative decay time, τ_{rad}, and this expression can then be simplified into

$$\frac{N_3 - N_2}{N} \approx \frac{(1-\beta)\eta.W_p.\tau_{rad}}{1 + [1 + \beta]\eta.W_p.\tau_{rad}} \approx \frac{W_p.\tau_{rad}}{1 + W_p.\tau_{rad}} \qquad \text{if } \beta \rightarrow 0 \tag{5.17}$$

The optimum situation is obviously $\beta \approx \tau_{21}/\tau_{32} \rightarrow 0$.

Figure 5.5 shows a plot of the inversion $N_3 - N_2$ on the four-level laser transition verses the

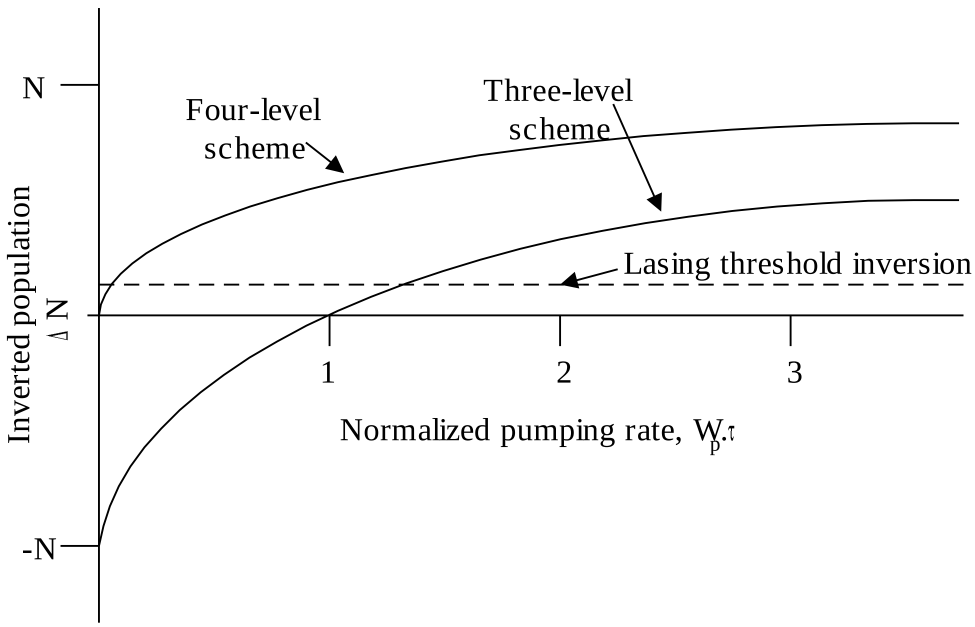


Figure 5.5 Laser population inversion verses normalized pumping rate for idealized four-level and three-level laser systems.

normalized pumping rate $W_p.\tau$, assuming β=0. For a four-level system, the population inversion on the $3 \rightarrow 2$ transition increases linearly with the pumping rate W_p at lower pump levels, but then approaches a limiting value for $W_p.\tau >> 1$ as the ground state E_1 is depleted and a large fraction of the atoms are lifted into the upper laser level.

This four-level pumping model provides an excellent analytical model for understanding the behavior of a large number of real laser systems.

5.3.2 Three-Level Laser System

Figure 5.6 illustrates how a three-level laser system can be similarly employed as a model for the real energy levels of the famous ruby laser. Similar to the model of four-level system for Nd:YAG, a model for three-level system can be formed.

A three-level system differs from the four-level system in that the lower laser level is the ground level E_1. This is a serious disadvantage, since more than half the atoms initially in the

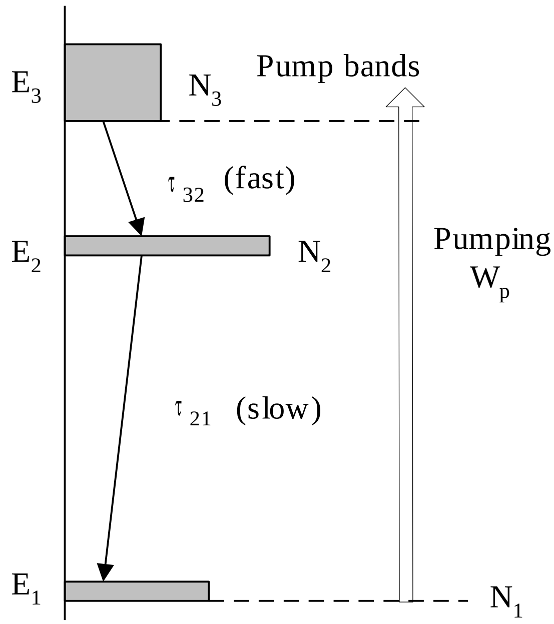


Figure 5.6 An idealized three-level model for laser pumping resembles closely with ruby laser system.

ground state must be pumped through the upper pumping level E_3 into the upper laser level E_2 before any inversion at all is obtained on the $2 \rightarrow 1$ transition. Three-level lasers are, therefore, usually not as efficient as four-level lasers. One reason for analyzing the three-level system, in addition to the general background knowledge, is that the 694 nm ruby laser, the first ever to be operated and still a useful solid state laser, is a nearly ideal three-level laser system.

Suppose the pumping process in a three-level system produces a stimulated transition probability $W_{13}=W_{31}=W_p$. Then the rate equations for the two upper levels are

$$\frac{dN_3}{dt} = W_p.(N_1 - N_3) - \frac{N_3}{\tau_3} \quad (5.18)$$

and

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} \quad (5.19)$$

There is also the usual conservation equation $N_1 + N_2 + N_3 = N$, and it is again useful to define fluorescence quantum efficiency given by

$$\eta = \frac{\tau_3}{\tau_{32}} \times \frac{\tau_2}{\tau_{rad}(2 \rightarrow 1)} \tag{5.20}$$

The important relaxation-time ratio in this model is given by

$$\beta = \frac{N_3}{N_2} = \frac{\tau_{32}}{\tau_{21}} \tag{5.21}$$

The steady-state population difference on the $2 \rightarrow 1$ transition can be found to be

$$\frac{N_2 - N_1}{N} = \frac{(1-\beta).\eta.W_p.\tau_{rad} - 1}{(1+2\beta).\eta.W_p.\tau_{rad} + 1} \tag{5.22}$$

Inversion in the three-level system can be obtained only if $\beta < 1$, and even then inversion can occur only when the pumping rate exceeds a threshold value given by

$$W_p \tau_{rad} \geq \frac{1}{\eta.(1-\beta)} \tag{5.23}$$

The optimum situation occurs when the relaxation from the energy level 3 into the upper laser level 2 is very fast, so that $\beta \rightarrow 0$, and when the relaxation from the upper laser level 2 down to the ground level 1 is purely radiative, so that $\eta \rightarrow 1$. The inversion verses normalized pumping strength then reduces to

$$\frac{N_2 - N_1}{N} \approx \frac{W_p.\tau_{rad} - 1}{W_p.\tau_{rad} + 1} \qquad \text{If } \eta \rightarrow 1 \text{ and } \beta \rightarrow 0 \tag{5.24}$$

The significant differences in inversion verses pumping for a three-level and a four-level system are illustrated in the Figure 5.5. A four-level laser system should have a much lower pumping threshold than a three-level laser system.

***Example 1**. It is possible to have a three-level laser system which is pumped on the $1 \rightarrow 3$ transition and in which cw laser action takes place on the $3 \rightarrow 2$ rather than the $2 \rightarrow 1$ transition (no such real system is known). Suppose that level 3 in such a system is long lived with lifetime τ_3; level 2 has a short relaxation time to the ground state; and the system is pumped with transition probability W_p on the $1 \rightarrow 3$ transition.*

Derive the expression for population inversion between level 3 and 2.

***Solution**: Consider a three-level system as shown in Figure 5.7. The rate equations for the population of upper laser level E_3, and lower laser level E_2 are:*

Suppose the pumping process in a three-level system produces a stimulated transition probability $W_{13} = W_{31} = W_p$. Then the rate equations for the two upper levels are

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - \frac{N_3}{\tau_3} \tag{5.a}$$

and

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} \tag{5.b}$$

In steady-state condition the rate of change of atoms in the two energy levels will be zero, thus

$$\begin{aligned} \frac{dN_3}{dt} &= W_p(N_1 - N_3) - \frac{N_3}{\tau_3} = 0 \\ \Rightarrow N_3 &= \left(\frac{W_p.\tau_3}{1 + W_p.\tau_3} \right).N_1 \end{aligned} \tag{5.c}$$

similarly

and

$$\begin{aligned} \frac{dN_2}{dt} &= \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} = 0 \\ \Rightarrow N_2 &= \frac{\tau_{21}}{\tau_{32}}.N_3 \end{aligned} \tag{5.d}$$

From equation (5.c) and (5.d), we have

$$N_3 - N_2 = N_3 - \frac{\tau_{21}}{\tau_{32}} N_3 = \left(1 - \frac{\tau_{21}}{\tau_{32}}\right) \cdot \left[\frac{W_p.\tau_3}{1 + W_p.\tau_3}\right] . N_1 \qquad (5.e)$$

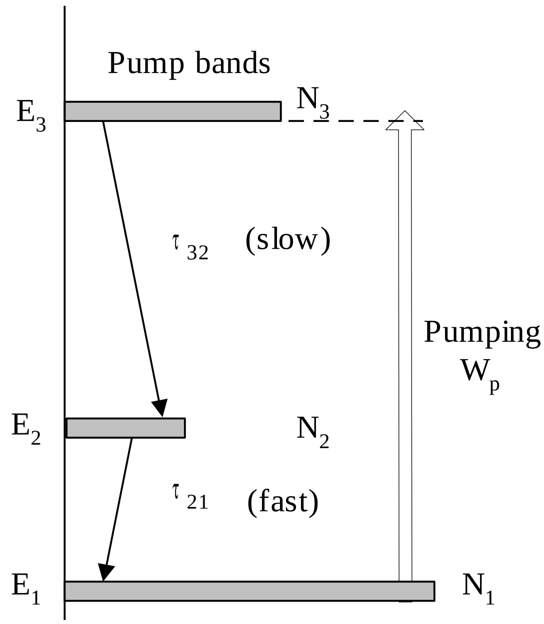


Figure 5.7 Three-level pumping scheme in which population inversion can be build-up between level 3 and level 2

The conservation of total number of atoms gives that

$$N_1 + N_2 + N_3 = N \qquad (5.f)$$

Therefore, we have

$$N = N_1 + \frac{\tau_{21}}{\tau_{32}} \left[\frac{W_p.\tau_3}{1 + W_p.\tau_3}\right] . N_1 + \left[\frac{W_p.\tau_3}{1 + W_p.\tau_3}\right] . N_1$$

$$N = \left[1 + \left(1 + \frac{\tau_{21}}{\tau_{32}}\right) \cdot \left(\frac{W_p.\tau_3}{1 + W_p.\tau_3}\right)\right] . N_1 \qquad (5.g)$$

Now from equations (5.e) and (5.g) we get

$$\frac{N_3 - N_2}{N} = \frac{\left(1 - \frac{\tau_{21}}{\tau_{32}}\right) \cdot \left[\frac{W_p . \tau_3}{1 + W_p . \tau_3}\right]}{1 + \left(1 + \frac{\tau_{21}}{\tau_{32}}\right) \cdot \left[\frac{W_p . \tau_3}{1 + W_p . \tau_3}\right]} \qquad (5.h)$$

let $\beta = \frac{\tau_{21}}{\tau_{32}}$*, then above equation can be written as*

$$\frac{N_3 - N_2}{N} = \frac{(1 - \beta).W_P . \tau_3}{(2 + \beta).W_P . \tau_3 + 1} \qquad (5.j)$$

In this kind of three-level laser, the fluorescence efficiency, η*, can be defined as*

$\eta = \frac{\tau_3}{\tau_{rad}}$*, the last equation can be written as*

$$\frac{N_3 - N_2}{N} = \frac{(1 - \beta).W_P.\eta.\tau_{rad}}{(2 + \beta).W_P.\eta.\tau_{rad} + 1} \qquad (5.k)$$

Population inversion in the given three-level laser system will be obtained if $\beta<1$ *and* $\eta\sim1$*. Therefore, substituting* $\beta \to 0$ *and* $\eta \to 1$ *in the above equation.*

$$\frac{N_3 - N_2}{N} = \frac{W_p . \tau_{rad}}{2.W_p . \tau_{rad} + 1} \qquad (5.l)$$

The above equation shows that population inversion is 0 at zero pumping rate and inversion can be achieved with a small pumping rate, that is, lasing threshold is not as high as in the case of ruby laser, but unfortunately, no such laser exist.

Example 2. *Consider a five-level atomic system in which the optical frequency approximation applies between all levels. Assume that this system is pumped on the* $1 \to 5$ *transition with a pumping transition probability* $W_{15} = W_{51} = W_p$*, and that each upper level in the system relaxes only into the level immediately beneath it. Evaluate the population difference on the* $3 \to 2$ *transition as a function of* W_p*.*

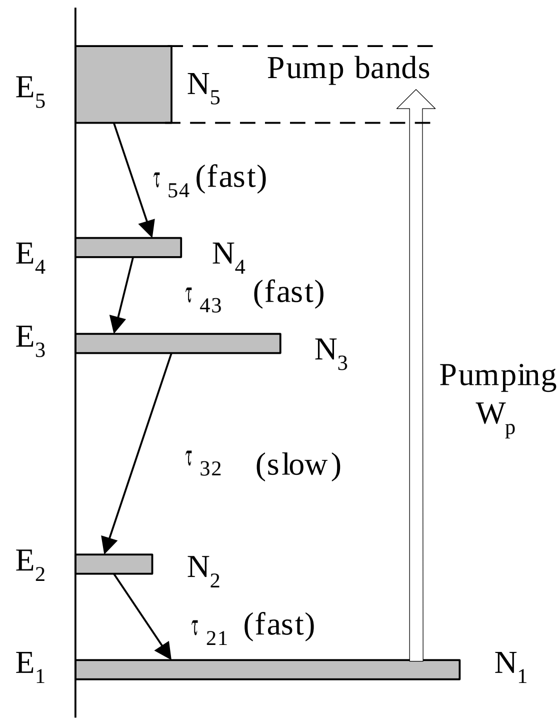


Figure 5.8 Five-level scheme of energy levels for pumping.

Solution: *Energy level diagram for the five-level atomic system is shown in the Figure 5.8. In the optical frequency approximation, the thermally stimulated terms in the relaxation rates are totally negligible compared to the spontaneous rates. This is due to the fact that Boltzmann ratio at optical frequencies is very small, on the order of 10^{-36} at room temperature.*

The rate equation for level 5 with pumping transition probability W_p, at steady-state, is given by (it is given that all the levels can relax only into the level immediately beneath it, therefore, relaxation from $5 \rightarrow 4$ is only considered)

$$\frac{dN_5}{dt} = W_p.N_1 - \frac{N_5}{\tau_{54}} = 0 \qquad (5.m)$$

$$\Rightarrow N_5 = W_p.\tau_{54}.N_1$$

similarly the steady-state population of levels 4, 3, and 2 are given by

$$\frac{dN_4}{dt} = \frac{N_5}{\tau_{54}} - \frac{N_4}{\tau_{43}} = 0 \quad , \qquad (5.n)$$
$$\Rightarrow N_4 = \frac{\tau_{43}}{\tau_{54}}.N_5 = W_p.\tau_{43}.N_1$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_{32}} = 0 \qquad (5.o)$$
$$\Rightarrow N_3 = \frac{\tau_{32}}{\tau_{43}}.N_4 = W_p.\tau_{32}.N_1$$

and

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} = 0 \qquad (5.p)$$
$$\Rightarrow N_2 = \frac{\tau_{21}}{\tau_{32}}.N_3 = W_p.\tau_{21}.N_1$$

From equations (5.o) and (5.p),

$$N_3 - N_2 = (\tau_{32} - \tau_{21}).W_p.N_1 \qquad (5.q)$$

The conservation condition for total number of atoms gives that

$$N = N_1 + N_2 + N_3 + N_4 + N_5$$

or

$$N = \{1 + (\tau_{21} + \tau_{32} + \tau_{43} + \tau_{54})W_p\}.N_1 \qquad (5.r)$$

From (5.q) and (5.r), we have

$$\frac{N_3 - N_2}{N} = \frac{(\tau_{32} - \tau_{21}).W_p}{1 + (\tau_{21} + \tau_{32} + \tau_{43} + \tau_{54})W_p} \qquad (5.s)$$

The condition of population inversion depends mainly on the lifetimes of the level 3 and level 2, and to achieve population inversion, the lifetime of the level 3, τ_{32} should *be larger that τ_{21}. If all the lifetimes are very small as compared to the τ_{32}, then equation (5.s) can be reduced as*

$$\frac{N_3 - N_2}{N} = \frac{\tau_{32}.W_p}{1+\tau_{32}.W_p} \qquad (5.t)$$

in this case population inversion can easily be achieved.

5.4 Laser Gain Saturation

The steady-state gain is a decreasing function of the cavity photon flux. Physically, the decrease of the gain with increasing photon flux is due to the fact that a large cavity photon number, and therefore a large stimulated emission (and absorption) rate, tends to equalize the populations of upper and lower laser levels. In this case the gain is saturated. The gain saturation plays a very important role in determining the output power of a laser.

The gain saturation can be understood by considering the microscopic behavior. The spontaneous and stimulated emissions apply to an atom in the upper laser level and cause it to drop to the lower level and emit a photon. Once it is in the lower level, it can decay further or it can absorb a photon back from the field. The lower level's absorption rate is exactly equal to the upper level's stimulated emission rate. When the field is so strong that these rates are much greater than the levels decay rate, the atom jumps so rapidly between the upper and lower levels that it has effectively the same probability of being in one or the other of these levels. Then the atom is equally often an absorber and an emitter, and in this extreme limit the gain is zero. Thus in general the gain coefficient of the medium must be reduced as the cavity photon number is increased.

5.4.1 Laser Gain Saturation Analysis

In many real laser systems laser action takes place between two excited levels that are located high above the ground levels. The population density in these excited lasers levels always remains small compared to the total population density of atoms in the lowest energy level E_1. This is particularly true in gas lasers, where line widths are narrow, transitions are relatively strong, and only small inversion densities are necessary to give sufficient gain. It is not true for solid-state lasers, for example, ruby laser, where large fraction of the total atomic density may sometimes be pumped into the upper laser levels.

In this section we will use a simplified model to develop some rate equations analysis, showing how the laser gain itself saturates with increasing signal power in typical laser systems.

Figure 5.9 gives a simplified but a realistic model for many laser systems. Atoms are pumped by some pumping mechanism from the ground level E_1 into some upper level E_4. They then relax down into the upper laser level E_3, from where they relax or make stimulated laser transitions down to the lower laser level E_2, and hence back to the ground level. In this analysis we have included a laser signal, corresponding to laser amplification or oscillation, and represented transition probability W in the diagram.

Suppose the upper-level populations all remain small compared to the initial ground-state population. Then the pumping rate from the ground level E_1 into the upper atomic level E_4 caused by a pumping transition probability $W_{14}=W_{41}=W_p$ may be written as

$$\left(\frac{dN_4}{dt}\right)_{pump} = W_p(N_1 - N_4) \approx W_p N_1 \tag{5.25}$$

where $N_1 \approx N$ is nearly equal to the total density of laser atoms in the system.

In this situation there is essentially no back pumping from E_4 to E_1, since very few atoms accumulate in the upper levels and hence $N_4 << N_1$. It is then convenient to speak of a net pumping rate (atoms per second per unit volume) being lifted up out of the ground level, as given by $W_pN_1 \approx W_pN$.

This pumping rate W_pN_1 in a real laser system will be more or less directly proportional to the pump light intensity (in an optically pumped laser), or the discharge current density (in a discharge-pumped gas laser). Moreover, in many real lasers some fixed fraction η_p of the atoms pumped into an upper energy level will decay, often through some cascade process, down into the longer-lived upper laser level E_3. The number of atoms per second reaching the upper laser level is then given by an effective pumping rate, $R_p=\eta_p.W_p.N_1$, where η_p represents the quantum efficiency for pump excitation into this upper

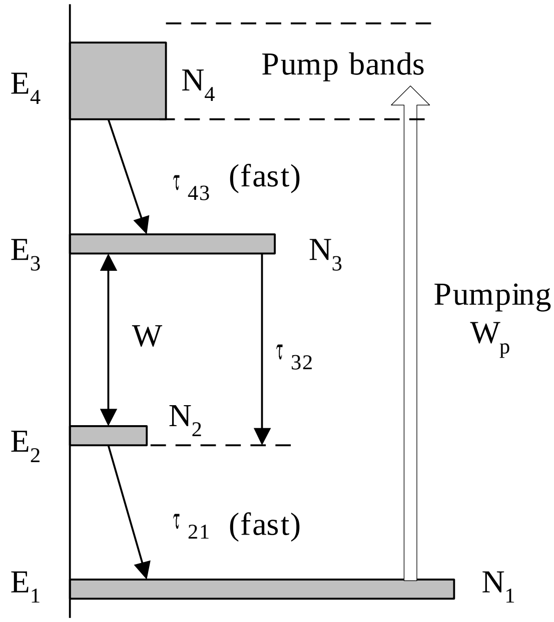


Figure 5.9 Simplified model for laser pumping and gain saturati between two high lying energy levels E_2 and E_3.

laser level. This pumping efficiency may be quite high, even approaching unity (for some lasers), for many solid state and organic dye laser, and may be very small for many typical discharge-pumped gas lasers.

With these generally valid assumptions, the rate equations for the excited laser levels E_3 and E_2, including stimulated transition probabilities $W_{23} = W_{32} = W$ on the laser transition, may be written as

$$\frac{dN_3}{dt} = R_p - W(N_3 - N_2) - \frac{N_3}{\tau_3} \tag{5.26}$$

and

$$\frac{dN_2}{dt} = W(N_3 - N_2) + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_2} \tag{5.27}$$

where τ_3 is the total lifetime of the upper laser level and

$$\frac{1}{\tau_3} = \frac{1}{\tau_{32}} + \frac{1}{\tau_{31}}. \tag{5.28}$$

The steady-state solutions to these equations are given by

$$N_2 = \frac{W + 1/\tau_{32}}{W(1/\tau_2 + 1/\tau_{31}) + 1/\tau_2.\tau_3}.R_p \tag{5.29}$$

$$N_3 = \frac{W + 1/\tau_2}{W(1/\tau_2 + 1/\tau_{31}) + 1/\tau_2.\tau_3}.R_p \tag{5.30}$$

The steady-state population difference $\Delta N = N_3 - N_2$ on the laser transition is given by

$$\Delta N \equiv N_3 - N_2 = \left(1 - \tau_2/\tau_{32}\right)\tau_3.\frac{R_p}{1 + \left(1 + \tau_2/\tau_{31}\right)\tau_3.W} \tag{5.31}$$

If the upper laser level E_3 relaxes primarily into the lower laser level E_2 and not directly down to any lower levels E_1, i.e. $\tau_3 \sim t_{32}$, then the above expression for population reduces to simply

$$\Delta N = R_p.\left(\tau_3 - \tau_2\right)\frac{1}{1 + \tau_3.W} \tag{5.32}$$

The inverted population difference in this example varies with both pumping rate and signals intensity in the opposite manner. The second term in the equation represent saturation behavior. As we increase the pump rate, R_p, the signal intensity increases and hence the stimulated emission probability W. At one stage it is the dominant mechanism to bring down the upper laser level and hence responsible for decrease the population inversion, thus reducing the gain. Therefore, at the saturation level gain is not proportional to the pumping rate, R_p.

Problems

5.1 Describe briefly:

(a) Why optical pumping is preferred for solid-state lasers and not for gas lasers?

(b) What is optical frequency approximation?

(c) Role of helium in He-Ne laser.

(d) Laser gain saturation.

5.2 Consider a three-level atomic system similar to one shown in Figure 5.6 with the difference that level 3 can decay to level 2 as well as to level 1 with life times τ_{32} and τ_{31} respectively. Calculate total lifetime τ_3, if τ_{31} = 3 ms, and τ_{32} = 300 μs. Is τ_3 is less than 300 μs? Give reasons.

5.3 Discuss the rate equations of a four-level system and derive expression of population inversion in the following situations:

(a) $(N_4-N_3)/N_1$ for τ_4 is large and τ_3 and τ_2 are very short.

(b) $(N_2-N_1)/N_1$ for τ_4 & τ_3 are very short and τ_2 is large.

5.4 Consider a four-level system in which the optical frequency approximation applies between all levels. Assume that this system is pumped from level1 to level 4 with transition probability $W_{14} = W_{41} = W_p$ and each upper level in the system relaxes to all lowering levels. Evaluate population difference on $3 \rightarrow 2$ transition as a function of W_p and relaxation times. What will be effect on population inversion when there is significant decay from level 4 to level 2, i.e., τ_{42} is not very long?

5.5 Suppose a four-level system is pumped with two separate pumping transition probabilities $W_{13} = W_{31} = W_A$ and $W_{34} = W_{43} = W_B$. The optical frequency approximation applies for all levels. Solve for the population difference N_4-N_2 in this system as a function of the two pumping powers WA and WB. Discuss what conditions are needed for an inversion on the 4-2 transition and how this inversion depends on the two pumping powers?

5.6 For a three-level laser system (like the ruby laser discussed in the text) assume for simplicity that the only relaxation processes present are τ_{32}, which is very fast, and τ_{21}, which is slow and purely radiative. In addition to a pump transition probability (W_p) on the $1 \rightarrow 3$ transition, add a signal transition probability W_{sig} on the $2 \rightarrow 1$ transition. Analyze the steady-state population inversion on the $2 \rightarrow 1$ transition as a function of W_p and W_{sig}.

Books for further reading

A. E. Seigman, *Lasers,* (University Science Books, California, 1986).

O. Svelto, *Principles of Lasers*, 3rd ed. (Plenum Press, New York, 1989)

P. W. Milonni and J. H. Eberly, *Lasers,* (John Wiley & Sons, New York, 1991).

Unit-6

Optical Resonators and Laser Modes

Objective

The cavity resonators used for optical radiation usually consist of two flat or curved mirrors set up facing each other, so that an optical wave can bounce back and forth between the two mirrors. Optical resonators of this type have many features in common with lens waveguide in which light is transmitted through a repeated series of identical lenses mounted in a line. In this unit we will discuss the important features of optical resonators. The laser output beam has some frequency structure due to the optical resonator, which are called laser modes. After the resonator, laser modes have been discussed in this unit.

6.1 Introduction

The laser is more analogous to an oscillator than an amplifier. In an electronic oscillator an amplifier which is tuned to a particular frequency is provided with positive feedback. The amplified output is fed back to the input and amplified yet again and so on. A stable output is quickly reached, however, since the amplifier saturates at high input voltages, as it cannot produce a larger output than the supply voltage.

In the laser, positive feedback may be obtained by placing the gain medium between a pair of mirrors which form an optical cavity or resonator. The initial stimulus is provided by any spontaneous transitions between appropriate energy levels in which the emitted photon travels along the axis of the system. The signal is amplified by the process of stimulated emission, as it passes through the medium and ‘fed back’ by mirrors. Saturation is reached when the gain provided by the medium exactly matches the losses incurred during a complete round trip.

The gain per unit length of most active media is so small that very little amplification of a beam of light results from a single pass through the media. In the multiple passes, which a beam undergoes when the medium is placed within a cavity, the amplification may be substantial.

We have quietly assumed that the radiation within the cavity propagates to and fro between two plane-parallel mirrors in a well-collimated beam. Because of diffraction effects, however, this cannot be the case as a perfectly collimated beam cannot be maintained with mirrors of finite size, some radiation will spread out beyond the edges of the mirrors. Diffraction losses of this nature can be reduced by using concave mirrors. In practice a number of different mirror curvatures and configuration are used depending on the applications and types of laser being used.

The commonly used mirror configurations are:

6.1.1 Symmetric Resonators: In symmetric resonators, both of the cavity mirrors have exactly same curvatures.

Plain-parallel configuration*: This configuration makes maximum use of the laser medium (i.e., we say mode volume is large) but it is difficult to align correctly.*

Confocal configuration: In this configuration the radii of curvatures of the two mirrors is equal to the length of the resonator. The confocal arrangement is easy to align but the only a fraction of active medium is used for lasing action (i.e., mode volume is small). In gas lasers, if maximum output power is required, we use a large radius resonator.

Concentric configuration: In this configuration two identically curved spherical mirrors are separated by a distance equal to twice their radius of curvature, therefore, their centres of curvature coincide.

6.1.2 Hemispherical Resonators: This configuration has combination of plane and curved mirrors in the resonator. If uniphase operation, i.e., maximum beam coherence is required, this configuration is preferred.

6.1.3 Unstable Resonator: In unstable resonators light rays diverge away from the axis. There are many variations in unstable resonators. One simple example is a convex spherical mirror opposite a flat mirror. Others include concave mirrors of different diameters and radii of curvatures.

Optical resonators can be studied using stability diagram. The design of an optical cavity can allow slightly different frequencies to get amplification in a resonator, which form the basis of laser modes.

6.2 Optical Resonators

A typical optical resonator formed by two curved mirrors with radii of curvature R_1 and R_2 spaced a distance L apart is shown in Figure 6.1. For each mirrors R > 0 implies that the

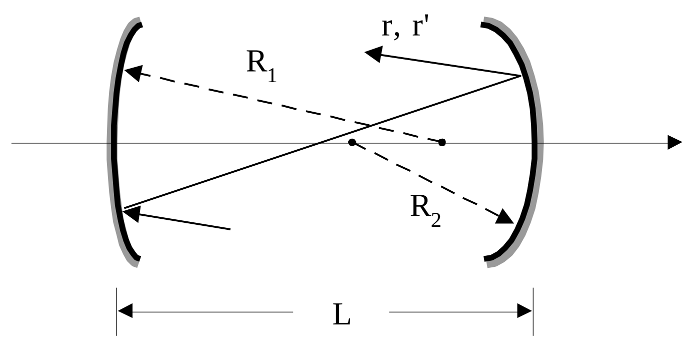


Figure 6.1 A typical optical resonator.

mirror is concave towards the resonator. In the discussion of ray matrices of optical components (Unit-3) we found that ray matrix of reflection from a spherical mirror is similar to the one for a lens. From this similarity (between a curved mirror and a thin lens) we can deduce that the behaviour of a ray upon repeated bounces back and forth between these two mirrors will be exactly the same as the behaviour of a ray passing through a series of lenses spaced at intervals L, with alternate focal lengths $f_1 = R_1/2$ and $f_2 = R_2/2$, as shown in Figure

6.2. The ray properties of the resonator in Figure 6.1 should be exactly same as the ray properties of the series of lenses in Figure 6.2.

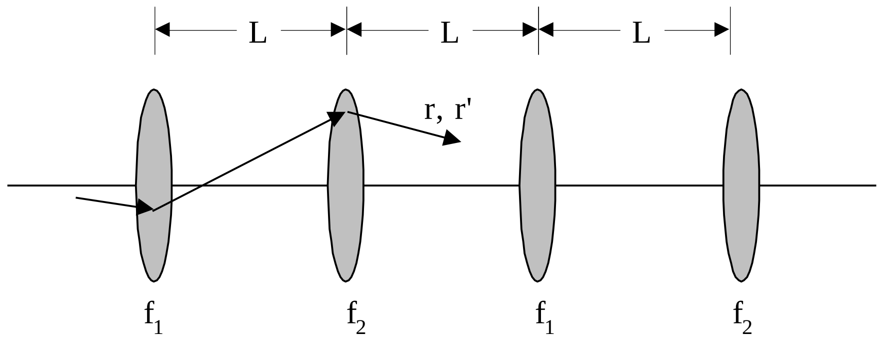


Figure 6.2 An equivalent peeriodic lens waveguide to an optical resonator.

6.2.1 Resonator 'g' Parameters

Let us define a new parameter 'g' which will be very convenient to describe the curvatures and spacing of a two mirror optical resonator, or the equivalent waveguide. The new parameter 'g' is defined as

$$g_1 \equiv 1 - \frac{L}{R_1}, \qquad (6.1a)$$

$$g_2 \equiv 1 - \frac{L}{R_2}. \qquad (6.1b)$$

The focal lengths of the two lenses f_1 and f_2 can be expressed in terms of the lens spacing L and the g parameters as:

$$f_1 = \frac{R_1}{2} = \frac{L}{2}\frac{1}{(1 - g_1)} \qquad (6.2a)$$

$$f_2 = \frac{R_1}{2} = \frac{L}{2}\frac{1}{(1 - g_2)} \qquad (6.2b)$$

Let us now analyse the behaviour of a ray upon repeated bounces in an optical resonator of Figure 6.1, or upon passing through a lens waveguide. We are interested to know whether after many bounces the ray will still be reasonably close to the axis of the system or whether it will diverge outward a large distance from the axis. If the ray remains close to the axis after multiple bounces, the resonator is said to be stable otherwise unstable.

We will first consider the net transformation of a ray in passing through one complete round trip inside the resonator or one full period of the series of periodic lens waveguide. It will simplify the analysis somewhat if we consider one full symmetric period of the system, such as the symmetric transformation from the mid-plane of one lens to the mid-plane of the next identical lens, as shown in Figure 6.3. When a ray passes through such a series of optical elements in cascade, the total or overall ray transformation of r and r` can be computed by successive application of the individual (ABCD) matrices, that is, successive multiplication of the ray vector by the individual ray matrices. If the procedure is applied to the sequence of elements shown in Figure 6.3, the resulting overall transformation for one full period (length 2L) is

$$\begin{bmatrix} r_1 \\ r_1' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/2f_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/f_2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1/2f_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_0 \\ r_0' \end{bmatrix} \qquad (6.3)$$

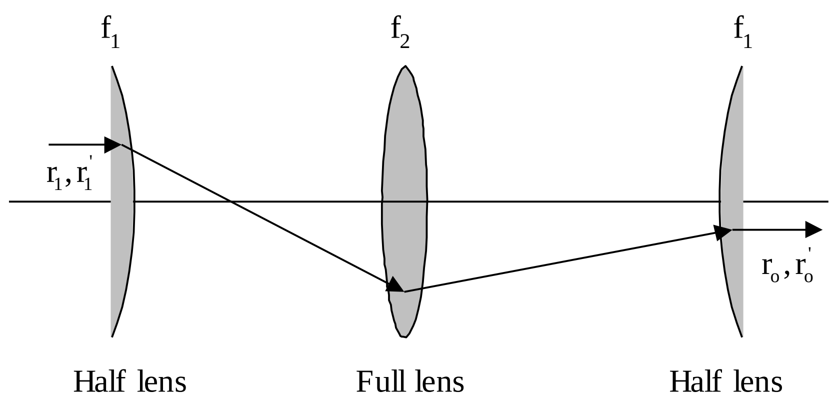


Figure 6.3 The basic unit for analysis, a single symmetric period of len wavegu extending from midplane of one lens to the midpoint of the next identical lens.

If the lens parameters are expressed in terms of g_1 and g_2, and the matrix multiplication in equation (6.3) are carried out, this expression for one complete bounce in the optical resonator, may be reduced to

$$\mathbf{r}_1 = \begin{bmatrix} r_1 \\ r_1' \end{bmatrix} = \begin{bmatrix} 2g_1g_2 - 1 & 2g_2 \\ 2g_1(g_1g_2 - 1) & 2g_1g_2 - 1 \end{bmatrix} \cdot \begin{bmatrix} r_0 \\ r_0' \end{bmatrix} = \mathbf{M}_{total}\, \mathbf{r}_0 \qquad (6.4)$$

We will try to find the eigenmodes of this system (say λ), i.e.; the output vector $\mathbf{r}_1$ may be obtained by multiplying the input vector $\mathbf{r}_0$ with λ. That is, we must determine input vector $\mathbf{r}_0$ and constant λ that will, together, exactly satisfy the requirement

$$\mathbf{r}_1 = \mathbf{M}_{total}\, \mathbf{r}_0 = \lambda \mathbf{r}_0$$

for this particular total ray matrix. This is equivalent to finding solutions to the matrix equation

$$\left(\begin{bmatrix} 2g_1g_2 - 1 & 2g_2 \\ 2g_1(g_1g_2 - 1) & 2g_1g_2 - 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right) \cdot \begin{bmatrix} r_0 \\ r_0' \end{bmatrix} = 0 \qquad (6.5)$$

The mathematical procedure for finding possible values of the constant λ, called the eigenvalues of the problem, consists of setting the determinant formed from the total matrix in equation (6.5) equal to zero,

$$\begin{vmatrix} (2g_1g_2 - 1) - \lambda & 2g_2 \\ 2g_1(g_1g_2 - 1) & (2g_1g_2 - 1) - \lambda \end{vmatrix} = 0 \qquad (6.6)$$

which gives the quadratic equation for the eigenvalues,

$$\lambda^2 - 2(2g_1g_2 - 1)\lambda + 1 = 0.$$

The two eigenvalues or solutions of this equation are then

$$\lambda = \lambda_a, \lambda_b = (2g_1g_2 - 1) \pm \sqrt{4g_1g_2(g_1g_2 - 1)} \qquad (6.7)$$

Along with these two eigenvalues λ_a and λ_b, there will be two eigenvectors $\mathbf{r}_a$ and $\mathbf{r}_b$, such that either will satisfy the basic eigenequation

$$\mathbf{M}_{total}\, \mathbf{r}_a = \lambda_a\, \mathbf{r}_a, \qquad \mathbf{M}_{total}\, \mathbf{r}_b = \lambda_b\, \mathbf{r}_b \qquad (6.8)$$

It is an important property of the eigenvectors $\mathbf{r}_a$ and $\mathbf{r}_b$ that they form a mathematically complete set: that is, any arbitrary input vector $\mathbf{r}_0$ may be written as a sum of $\mathbf{r}_a$ and $\mathbf{r}_b$ components in the form

$$\mathbf{r}_0 = C_a\, \mathbf{r}_a + C_b\, \mathbf{r}_b \qquad (6.9)$$

where C_a and C_b are expansion coefficients. Since passage through one section of the lens waveguide simply implies each eigenvector by its respective eigenvalue, therefore, the output ray after one section (or one complete round trip around the resonator) is given by

$$\mathbf{r}_1 = \lambda_a C_a \mathbf{r}_a + \lambda_b C_b \mathbf{r}_b \tag{6.10}$$

It is easy to calculate the output after an arbitrary number n of complete periods, since each eigenvector component is simply multiplied by its own eigenvalue to the nth power; i.e., the output is

$$\mathbf{r}_n = (\lambda_a)^n C_a \mathbf{r}_a + (\lambda_b)^n C_b \mathbf{r}_b \tag{6.11}$$

6.2.2 Stable Systems

If the g parameters satisfy the condition $0 \le g_1 g_2 \le 1$, the eigenvalues may be written as

$$\lambda_a, \lambda_b = (2g_1 g_2 - 1) \pm j\sqrt{4g_1 g_2 (1 - g_1 g_2)} \tag{6.12}$$

and this may also be written as

$$\lambda_a, \lambda_b = \cos\theta \pm j \cdot \sin\theta = e^{\pm j\theta} \tag{6.13}$$

where $j^2 = -1$ and $\theta = \cos^{-1}(2g_1 g_2 - 1) \qquad 0 \le g_1 g_2 \le 1$

The eigenvalues in this case are complex numbers with magnitude unity and phase angle $\pm\theta$. Since $\lambda^n = (e^{\pm j\theta})^n = e^{\pm jn\theta}$, propagation of an arbitrary initial ray through n periods of a lens waveguide, or through n bounces in an optical resonator, will lead to an output of the form

$$\mathbf{r}_n = C_a e^{jn\theta} \mathbf{r}_a + C_b e^{-jn\theta} \mathbf{r}_b = (C_a \mathbf{r}_a + C_b \mathbf{r}_b) \cos(n\theta) + j(C_a \mathbf{r}_a - C_b \mathbf{r}_b) \sin(n\theta) \tag{6.14}$$

The displacement of the ray after n periods, where n is an arbitrary integer, will be given by an expression of the form

$$\mathbf{r}_n = \mathbf{r}_0 \cos(n\theta) + \mathbf{S}_0 \sin(n\theta) \tag{6.15}$$

where S_0 is related to the initial ray slope. The displacement of a ray while passing through a lens waveguide is illustrated in Figure 6.4, the ray in this case oscillate back and forth in sinusoidal fashion about the system axis, with a maximum limit determined by the entrance condition $\mathbf{r}_0$ and r_0'. Thus the focussing system is stable in the sense that while a ray oscillates about the axis; it always remains within bounded maximum limits. Therefore, for g values satisfying the condition $0 \le g_1 g_2 \le 1$ the focusing system is stable.

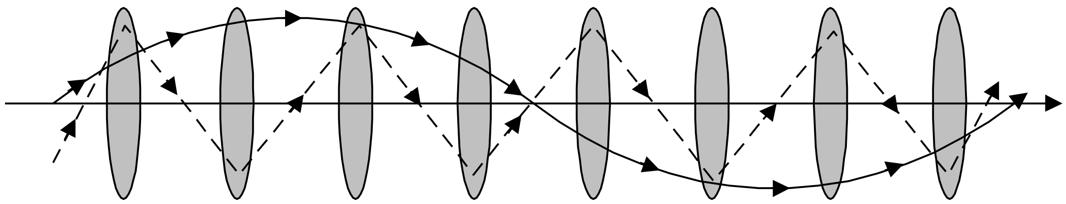


Figure 6.4 Ray propagation in a stable periodic-focussing system. The solid dashed lines show two different types of stable oscillatory behaviour depending the focussing properties of the lenses.

6.2.3 Unstable Systems

In optical resonators or lens waveguides with $g_1g_2<0$ or $g_1g_2>1$, the eigenvalues have the form

$$\lambda_a, \lambda_b = (2g_1g_2 - 1) \pm \sqrt{4g_1g_2(1 - g_1g_2)} \qquad (6.16)$$

which may be written as

$$\lambda_a, \lambda_b = e^{+\alpha a}, e^{-\alpha b} \qquad g_1g_2<0 \text{ or } g_1g_2>1 \qquad (6.17)$$

In contrast to the stable situation, the trajectory of an arbitrary ray in this case is shown in Figure 6.5. A ray in such a system will diverge exponentially with increasing number of sections or bounces and will eventually pass out of the system or intercept some limiting boundary of the system. The solid and dashed lines show two different forms of unstable behaviour.

6.2.4 Stability Diagram

The stability criterion developed from the ray-optics approach is basic and retains its validity even in more rigorous analysis. This stability criterion is

$$0 \le g_1 g_2 \le 1 \qquad \text{stable resonators} \qquad (6.18)$$

$g_1g_2<0$ or $g_1g_2>1$ unstable resonator. (6.19)

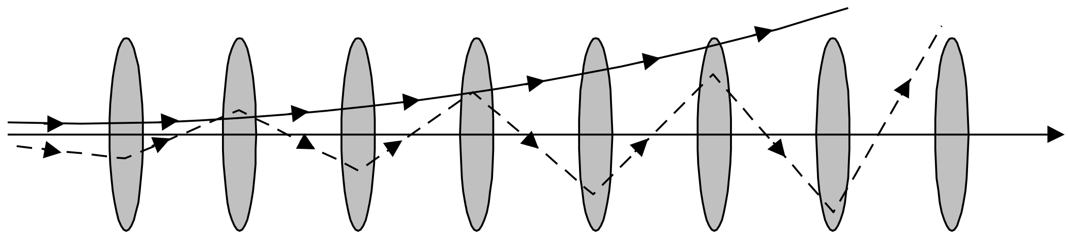


Figure 6.5 Ray propagation in an unstable system.

The criterion can be represented graphically by the stability diagram of Figure 6.6. Every two-mirror optical resonator can be characterised by the parameter g_1 and g_2, and hence represented by a point on the g_1g_2 plane. The resonator or focussing system will be stable only if this point falls within the shaded region of Figure 6.6. The planar, confocal, and

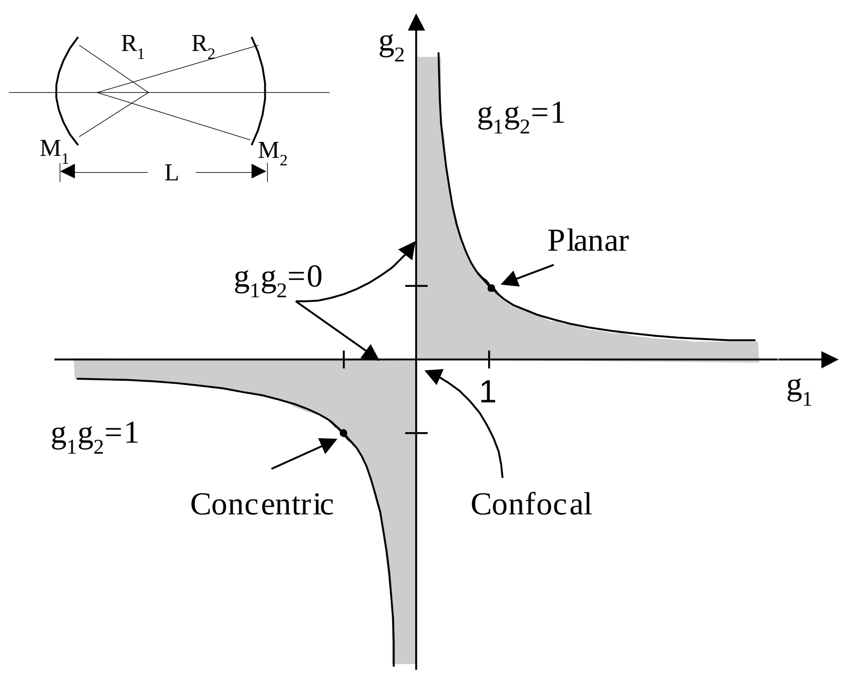


Figure 6.6 The stability diagram for optical resonators and lens waveguide; systems falling in between the axis and the $g_1g_2 = 1$ curve are stable.

concentric resonators are indicated specifically on the diagram. In Figure 6.7 the mirror curvatures appropriate to various regions have been overlaid on the stability diagram. It is interesting to note that a system with two convergent mirrors can still be unstable if the mirror convergence is too strong. Such a system is ‘overfocused’. The diagram also shows that it is possible in some cases to have a stable resonator with one divergent mirror, if the other mirror has the proper convergence to make the overall system stable. It is important to note that the stability and instability depend only on the g parameters, and are independent of either the optical wavelength or the transverse size or dimensions of the resonator. In the following section we will examine the various types of resonators that occur in various regions of the stability diagram.

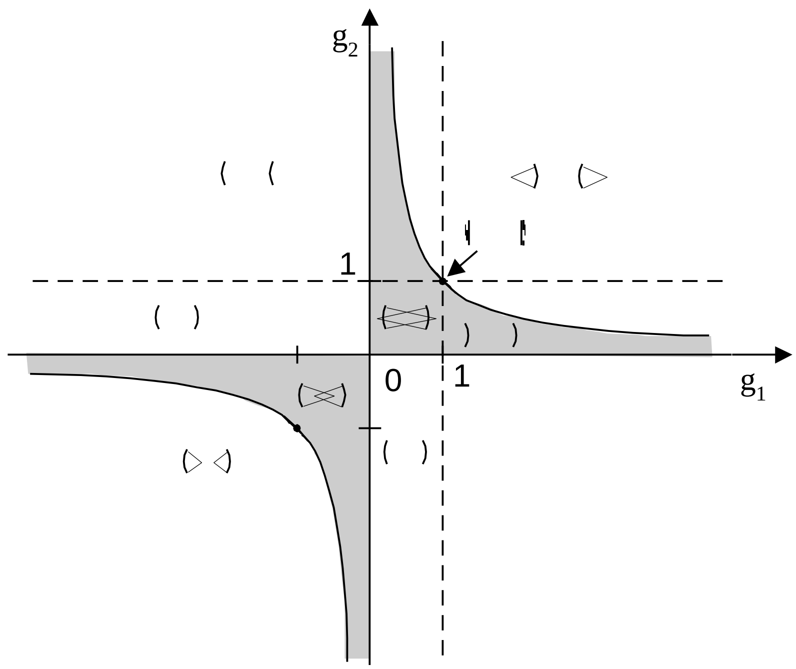


Figure 6.7 The examples of mirror configurations corresponding to differ regions of g_1g_2 plane.

6.3 Important Resonators Types

To gain more insight into the optical resonators, we will survey some resonators at various different points of interest in the stability diagram.

6.3.1 Symmetric Resonators

The simplest resonator configurations to analyse are symmetric resonators, which have mirror curvatures $R_1 = R_2 = R$, and hence g parameters $g_1 = g_2 = g = 1 - L/R$. These symmetric resonators lie along the +45^{o} diagonal through the origin in the g-plane, as shown in Figure 6.8. Three important categories in this configuration are:

1. symmetric confocal resonator, i.e., g = 0
2. two-planar mirror resonator, i.e., g = 1, and
3. concentric or spherical resonator, i.e., g = -1.

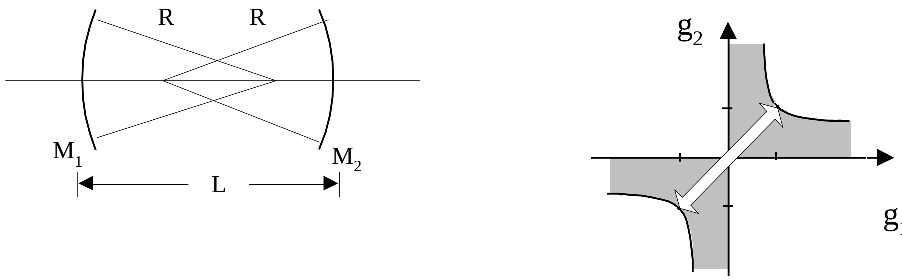


Figure 6.8 Symmetric stable resonators lie along the diagonal axis in the g_1g_2 plane.

6.3.1.1 Symmetric Confocal Resonator

The central point in the stability diagram and an important type of stable optical resonator is the symmetric confocal resonator. This resonator consists of two concave mirrors having radius of curvatures $R_1 = R_2 = L$ thus $g_1 = g_2 = 0$ (Figure 6.9). This is referred to as a confocal resonator because the focal points of the two end mirrors (which are located at the R/2 out from the mirror) coincide with each other at the centre of the resonator.

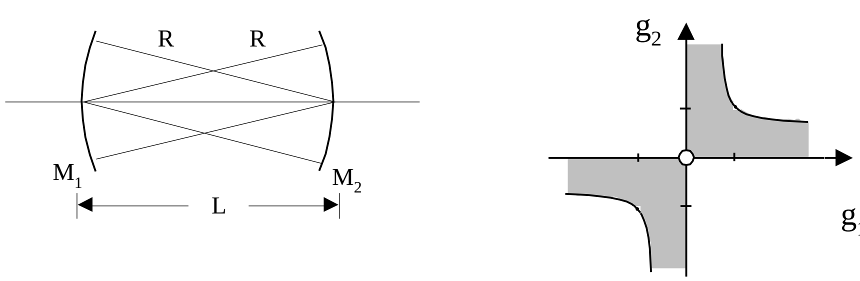


Figure 6.9 The symmetric confocal resonator is a special case, located exactl the centre of the stability diagram.

The confocal resonator is highly insensitive to misalignment of either mirror. Tilting of either mirror still leaves the centre of curvature located on the other mirror surface, and merely displaces the optic axis of the resonator by a small amount. The confocal can be very useful, for example, as a trial resonator design when we are first attempting to obtain laser oscillation from a laser medium whose gain is small or uncertain.

6.3.1.2 Long Radius (Near-Plane) Resonators

Another elementary resonator configuration, and one of that was used in many of the earliest laser devices, is the near-planar or long-radius stable resonator of Figure 6.10. A planar or flat-mirror resonator can be regarded as the limiting case of a long-radius stable resonator as the radii of curvature of the mirrors go to infinity. The resonator parameters then become $R_1 \approx R_2 \approx \infty$ and $g_1 \approx g_2 \approx 1$. The exactly planar resonator occurs right on the stability boundary, at $g_1 = g_2 = 1$.

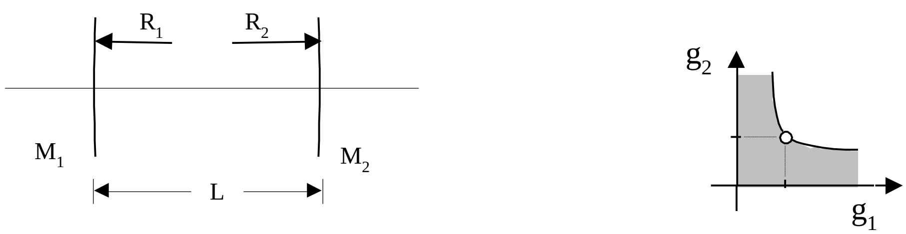


Figure 6.10 Long radius or near planer resonator can have larger mode volu
but is very sensitive to mirror allignment.

Although this configuration makes maximum use of the laser medium, however, the long radius resonators are generally avoided in practical laser designs because of their serious alignment difficulties. Long-radius mirrors are also difficult to manufacture and to test.

6.3.1.3 Near-Concentric Resonators

The near-concentric resonator is another design, which is on the boundary of the stability region. In this resonator the cavity length is less than the sum of the two radii R_1+R_2 by the small amount ΔL as illustrated by Figure 6.11. The resonator parameters are given by

$$R_1 \approx R_2 \approx R = L/2 + \Delta L \text{ and } g_1 \approx g_2 = -1+\Delta L/R. \qquad (6.20)$$

The mirror radii are physically reasonable and they can be pulled slowly apart in order to bring the resonator closer to or even across the stability boundary. The central portion of the resonator is not very useful, at least for laser power extraction. This resonator is also very sensitive to very small mirror misalignments.

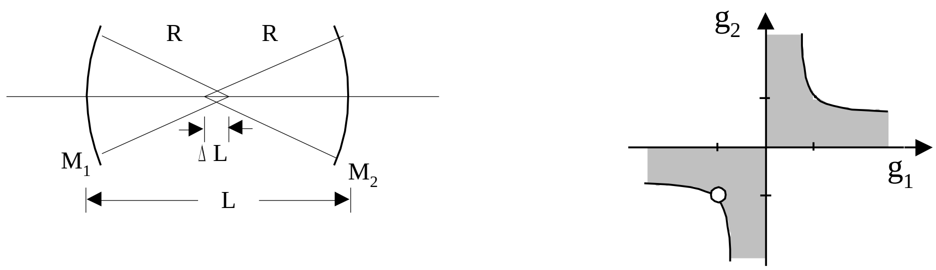


Figure 6.11 The near concentric resonator lies close to the $g_1 g_2$ = -1 point in the stability diagram.

6.3.2 Hemispherical Resonators

The resonator design that is commonly used in practical stable-resonator lasers (e.g., most medium and low-power gas lasers) is the near-hemispherical or half-concentric stable resonator, shown in Figure 6.12. The resonator parameters for this resonator are $R_1 = \infty$ and $R_2 = L + \Delta L$, and hence $g_1 = 1$ and $g_2 = \Delta L/L \approx 0$.

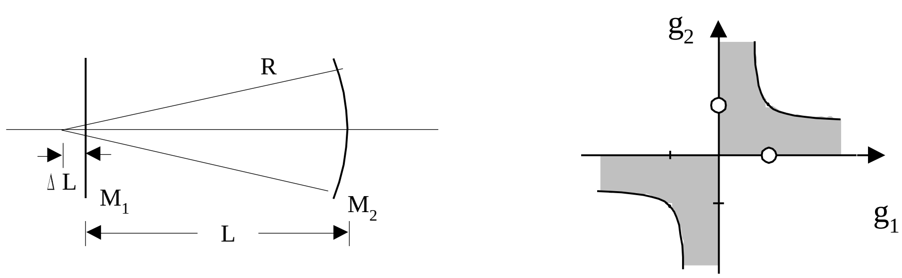


Figure 6.12 The near-hemispherical resonator is widely used in practical la[illegible] oscillators.

The useful volume in a near-hemispherical resonator is essentially in the shape of a cone. Lasers with near-hemispherical resonators are usually designed with the cavity somewhat longer than the active laser volume. The laser tube is placed near the large-diameter end of the cavity. In typical helium neon lasers, the discharge region is usually stopped well short of the flat-mirror end of the laser. The main advantage of the hemispherical design is that alignment difficulties are largely eliminated.

6.3.3 Concave-Convex Resonators

Any design which operate close to the stability boundary can give large volume size but at the expense of high sensitivity to small fluctuation in the mirror curvature or spacing. By moving out into the regions of the stability diagram beyond $g_1 = 1$ or $g_2 = 1$. It is possible to have so called concave-convex stable resonator such as illustrated in Figure 6.13. Resonators of this configuration have found some practical use, but generally tend to require inconveniently long mirror radii and sensitive alignment procedures.

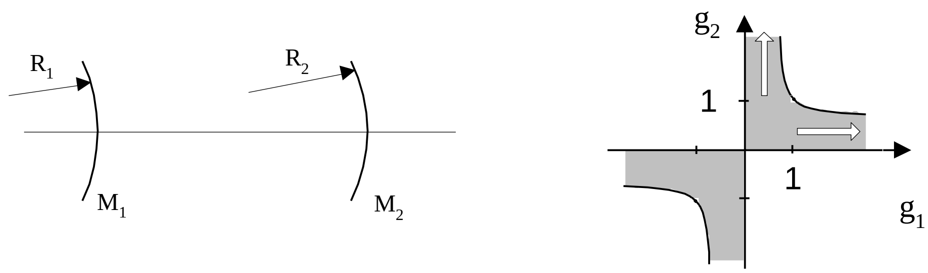


Figure 6.13 Concave-convex stable resonators can also provide large mc volumes, but are seldom used in practice.

6.3.4 Unstable Confocal Resonator

Finally, it is also possible to have resonators that are confocal but asymmetric i.e., resonator in which the two mirrors have different radii of curvature R_1 and R_2 but their focal points still coincide as in Figure 6.14. The spacing for a general asymmetrical confocal resonator is

$$R_1/2 + R_2/2 = L \qquad (6.21)$$

which can be translated into the condition

$$g_1 + g_2 = 2.g_1g_2. \qquad (6.22)$$

Examination of the stability diagram shows that this condition corresponds to a contour of locus which is unstable everywhere in the g_1, g_2 plane, except at the symmetric confocal point $g_1 = g_2 = 0$, and the planar symmetric point $g_1 = g_2 = 1$. Therefore, all asymmetric confocal resonators are unstable. The symmetric confocal resonator shown in Figure 6.9 is located at a kind of singular point in the stability diagram. A small deviation from this point in different directions can lead to either stable or unstable regions of the plane.

The emphasis on stable resonators does not imply that unstable resonators have no practical applications. On the contrary, unstable resonators enjoy certain advantages, and they are essential to the design of many high power lasers. In fact, stable resonators have some drawbacks at high power systems. A major disadvantage is that the modes of stable resonators tend to be concentrated in very thin regions within the resonator. Therefore they do not overlap a very large portion of the gain medium, and this presents a problem if high power extraction from the medium is desired.

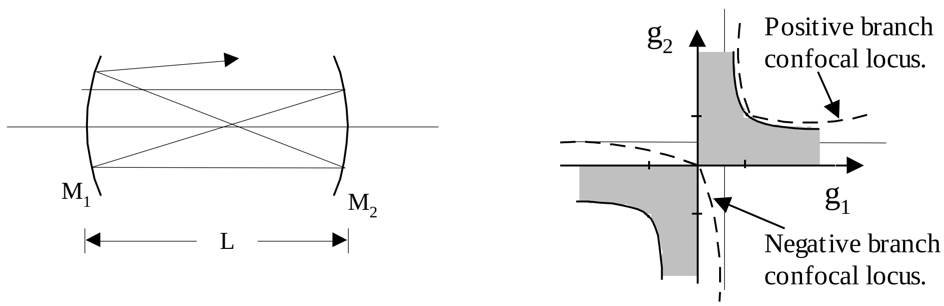


Figure 6.14 All asymmetric confocal resonators lie outside the shaded region a
are thus unstable.

Unstable resonators typically have much larger mode volumes, and can therefore make better use of the available gain region. Figure 6.15 shows an important practical example of an unstable confocal resonator (g_1g_2>1). The intra-cavity field fills a large portion of the cavity, and can be made larger simply by using larger mirrors. The output beam for resonator shown in Figure 6.15 is a collimated annular (doughnut-shaped) beam in the near field close to the resonator.

Unstable resonators offer other advantages in addition to their large mode volume. They can give higher output powers when operating on the lowest-loss transverse mode rather than on several modes. This is an important property and in stable resonators the situation is completely reverse. Another advantage is that unstable resonator lasers use all-reflective optics. That is, the output does not pass through any mirrors but simply spills around the edges. Therefore, at high power operation water cooling of the optics is easier as compare to transmission optics. Examples of a few configurations of unstable resonator are summarised in Figure 6.16.

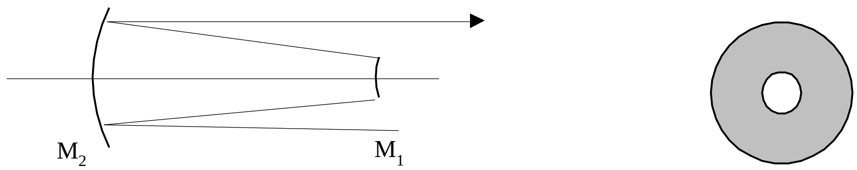


Figure 6.15 A positive branch (g_1g_2 >1) confocal redonator.

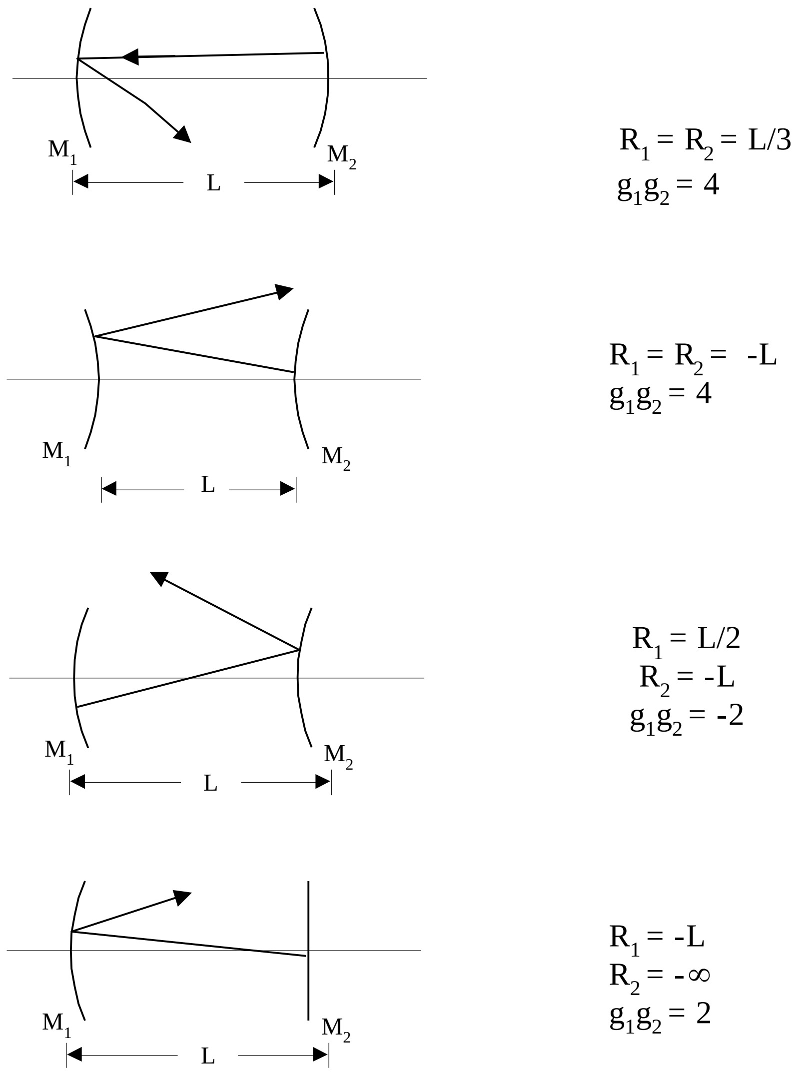


Figure 6.16 Examples of unstable resonators.

6.4 Laser Modes

Examination of a laser output with a spectrometer of very high resolving power, such as the scanning Fabry-Perot interferometer, can show that it consists of a number of discrete frequency components. To understand the reason of these discrete lines and its relationship with the laser transition lineshape we have to examine the effects of mirrors on the light within the laser cavity and the quality factor Q of the resonator. The factor Q can be defined in general by

Q=2π x the energy stored in the resonator / energy dissipated per cycles

or in lasers

$$Q = \text{resonant frequency / linewidth} = \frac{\nu}{\Delta\nu} \tag{6.23}$$

For an electrical oscillator Q may be approximately 100, whereas for a laser Q may be $\sim 10^8$. In lasers the active medium is actually supplying energy to the oscillating modes so that in theory the energy dissipation can be zero and Q infinite. In practice there are always losses, which prevent this happening.

6.4.1 Longitudinal Modes

The optical cavity of a laser is a resonator with extremely high Q and low losses. If these losses are smaller than the gain of the active medium, threshold is achieved and lasing occurs. But the high Q condition does not hold for all frequencies within the laser emission linewidth; only certain frequencies fulfil the resonance conditions. Thus the laser output spectrum does not resemble the spontaneous emission lineshape, but rather consists of a series of narrower lines corresponding to the high-Q frequencies of the laser cavity.

To determine the conditions for high Q in a laser, we start with a plane wave of light propagating along a line in between two parallel mirrors. The round trip distance for a wave undergoing reflection at the mirrors is 2L, twice the distance between mirrors. The total phase change, $\Delta\phi$, of the wave in travelling a full round trip is equal to the 2π times the number of waves in length 2L (i.e., $2L/\lambda$) is:

$$\Delta\phi = 2\pi\frac{2L}{\lambda} = \frac{4\pi L}{\lambda} \tag{6.24}$$

If the reflected wave is 180^o out of phase with the original wave and of equal magnitude then there is no net field and therefore no net energy output from the resonator. This is due to the fact that the wave has not replicated itself upon reflection. Only at such a frequency that the wave and its reflections are in phase ($\Delta\phi = 2\pi q$, q is an integer) does the wave replicate itself. With replication, the electric fields add in phase. The resultant energy density is sufficient to induce substantial stimulated emission at that frequency. In other words, the mirrors form a resonant cavity in which light energy may be stored by multiple reflections between them. If the waves are replicated in the cavity, then the mirror cavity has a high Q. The condition for a self-repeating field (setting $\Delta\phi = 2\pi q$ in the equation 6.24) is that the length of the cavity be equal to an integral number of half-wavelengths, or $L = q(\lambda/2)$, q an integer. Only at those wavelengths is the cavity resonant. The integer q in most cases quite large. For example, if the central wavelength is 500 nm and the mirror separation is 50 cm, q has a value of $2x10^6$. Since q can be any integer, there are many possible wavelengths within the laser transition lineshape for which the field is self-replicating. We refer to such self-replicating field pattern as longitudinal mode or axial mode of the cavity. It is easier to refer to these axial modes by their frequency than by wavelength. Using the condition for self-replicating field stated above, we have

$$\nu = \frac{c}{\lambda} = \frac{c}{(2L/q)} = q\frac{c}{2L} \qquad (6.25)$$

Each mode frequency can be labeled with its corresponding integer q, with the result

$$\nu_q = q\left(\frac{c}{2L}\right) \qquad (6.26)$$

It is at these frequencies that the laser cavity is resonant.

By subtracting the frequency of one cavity mode from its nearest neighbour, we find that the separation between mode frequencies is

$$\begin{aligned}\Delta\nu = \nu_{q+1} - \nu_q &= (q+1)\frac{c}{2L} - q\frac{c}{2L} \\ &= \frac{c}{2L}\end{aligned} \qquad (6.27)$$

The separation between longitudinal mode frequencies only depends on the mirror separation or cavity length, L, and is independent of q. If we use values from the previous example, the separation between the neighbouring resonance frequencies for a typical laser (50 cm long) is calculated to be

$$\Delta\nu = \frac{3\times10^{8}\,\mathrm{m/sec}}{2\times50\times10^{-2}\,\mathrm{m}} = 3\times10^{8}\,\mathrm{sec}^{-1} = 300\,\mathrm{MHz}. \tag{6.28}$$

Many laser transition lines are much broader than 300 MHz, and thus there can be many axial modes (…q-2, q-1, q, q+1, q+2, …) within the broadened linewidth. Since sustained laser action can occur only at those frequencies within the lasing transition for which the cavity is resonant, the output of laser contains a number of discrete frequencies, separated by (c/2L), as shown in Figure 6.17. These frequencies are called the axial mode frequencies of the laser.

It is interesting to know that while all the integers q give possible axial cavity modes only those which lie within the gain curve of the laser transition line will actually oscillate. The linewidth of laser transition (Figure 6.17), for the 632.8 nm wavelength, emitted by neon is about 1.5×10^{9} Hz wide. Therefore, with the 0.5 meter long cavity in the above example we would expect four or five modes to be present as illustrated in Figure 6.17c.

6.4.2 Longitudinal-Transverse Modes

In the previous section, we have restricted our attention to resonance conditions for plane waves travelling along a line joining the centres of the mirrors, i.e. along the optic axis of the cavity. For any real laser cavity, the complete wave pattern consists of a superposition of a large number of waves, each wave travelling in a slightly different direction, only nominally along the optic axis. Under these circumstances, the resonance condition is more complicated. The basic requirement is that the electromagnetic field distribution in the cavity replicate itself upon round-trip reflection by the mirrors. Because of their three dimensional

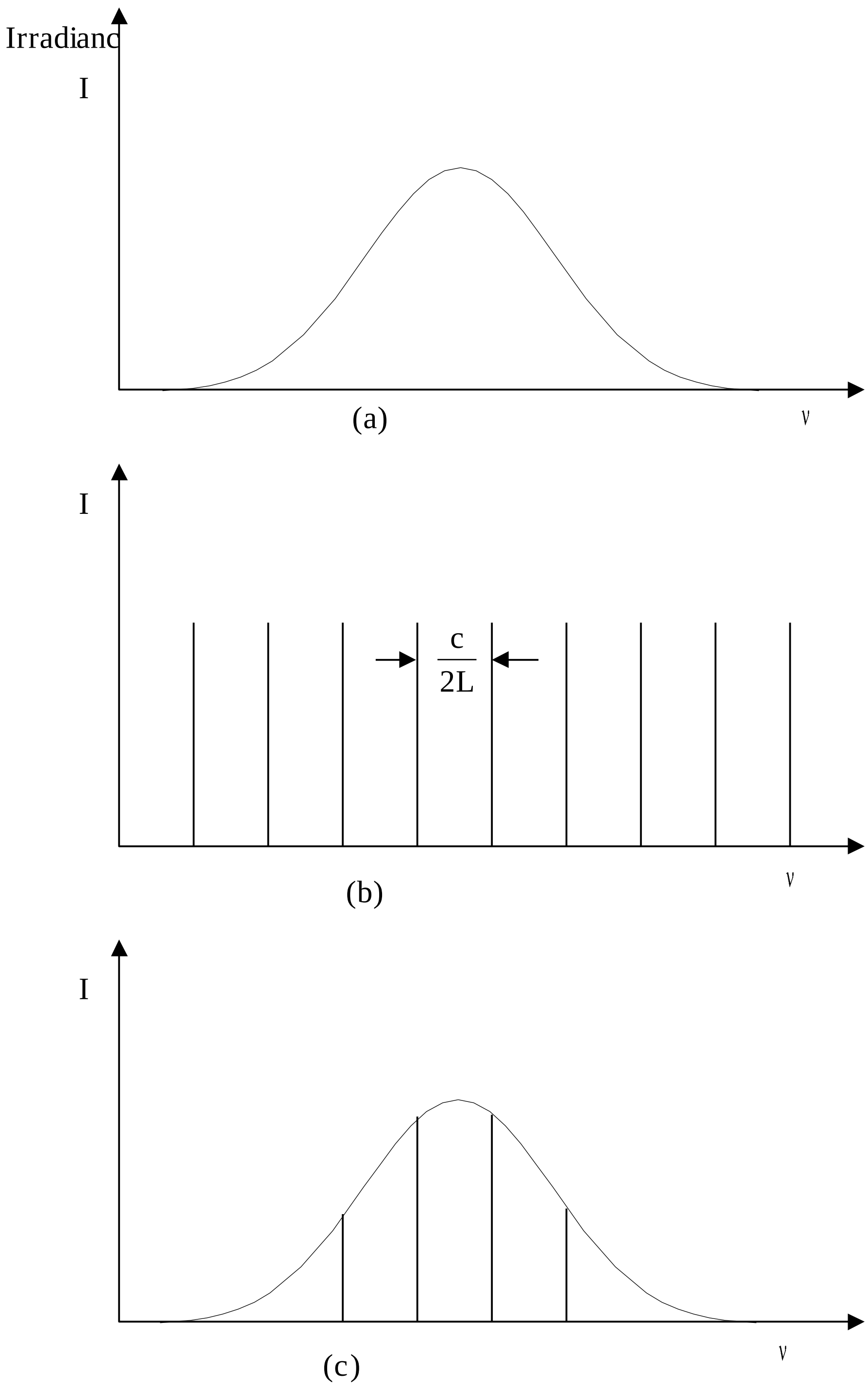


Figure 6.17. (a) The broadened laser transition, (b) cavity modes and (c) axial modes in the laser output.

nature, these self-replicating field pattern can be called as the longitudinal-transverse resonance modes of the laser cavity.

After considering the transverse character of the mode, it is no longer possible to characterize a cavity mode by a single number q. With the application of diffraction theory, it is possible to determine those field distributions that exhibit this self-replicating property in a laser cavity and to determine the frequencies of the corresponding modes. We simply quote the result using g parameters for optical cavities, the frequencies satisfying the full three-dimensional resonance condition can be expressed by the relationship

$$\nu_{mnq} = \left(q + (m+n+1) \frac{\cos^{-1}\sqrt{g_1 g_2}}{\pi} \right) \frac{c}{2L} \tag{6.29}$$

where m, n, and q are integers. This expression can be simplified for the common case of identical near-planar mirrors. Under these circumstances, $g_1 = g_2$ and $L/R << 1$ (here L is the length of the cavity and R is the radius of curvature of the end mirrors), we obtained the simpler expression,

$$\nu_{mnq} \cong \left(q + (m+n+1) \sqrt{\frac{2L}{R}} \right) \frac{c}{2L} \tag{6.30}$$

In both equations, the number q is associated with the axial character of the mode. On the other hand, m and n relate to the transverse mode number. Three mode numbers thus characterise a mode of a laser. In practice, the term “axial mode” is often used to differentiate between cavity modes with different values of q. In the same way; the term “transverse mode” is used to differentiate between modes of different m and n values. The different modes are designated by the notation TEM_{mnq}, where TEM denotes Transverse Electro-Magnetic, the light waves consisting of electromagnetic fields that are transverse to the direction of propagation. The value of q is quite large for practical laser dimensions. The values of m and n are usually quite small and often determined by an inspection of the laser output. As a consequence, in the labeling of modes with specific values of m, n, and q, the q is generally suppressed, with the resultant designation having the form TEM_{mn}. It is important to remember that although the mode designation does not contain q, each mode still retains a longitudinal character corresponding to some specific value of q. The important consideration is generally how many longitudinal modes (i.e., different values of q) are present in a laser output rather than the specific q-value, since the number of modes determine the total spread in the laser output spectrum.

Sophisticated equipment (e.g. Fabry-Perot interferometer) is required to observe the longitudinal character or mode structure of the laser output. But the transverse character can be easily identified by placing an inexpensive lens in the beam and observe the expanded beam on a screen. Typical pattern is illustrated in Figure 6.18. These transverse mode patterns depend only on m and n, not on q. The pattern associated with each mode are different and easily distinguished. The TEM^*_{10} mode pattern is a combination of the TEM_{01} and TEM_{10} patterns and may occur when there is a small lossy region or obstruction on the optic axis. It is also given a descriptive name as “doughnut mode” A self-replicating ray for TEM_{01} mode is shown in Figure 6.19. In lasers the most common mode is known as TEM_{00} mode, which has a Gaussian irradiance pattern and the smallest beam divergence of any of the modes.

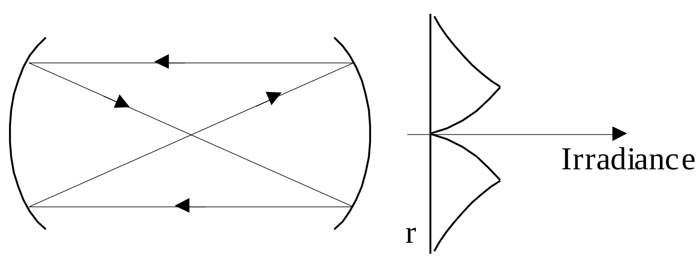


Figure 6.18. Formation of the TEM_{01} mode.

It is possible for a laser to operate in more than one transverse mode, just as it is possible for a laser output to contain more than one axial mode frequency. The expanded beam patterns are then a combination of the contributing separate patterns. It is possible that two modes may have the same q-value (i.e., identified with the same longitudinal mode), the frequencies can still be different.

How many lines can there be beneath the lasing transition? The number of longitudinal modes is determined by the linewidth of the transition line and by the length of the laser. The longer the laser, the smaller the separation between the modes of different q-values, and thus

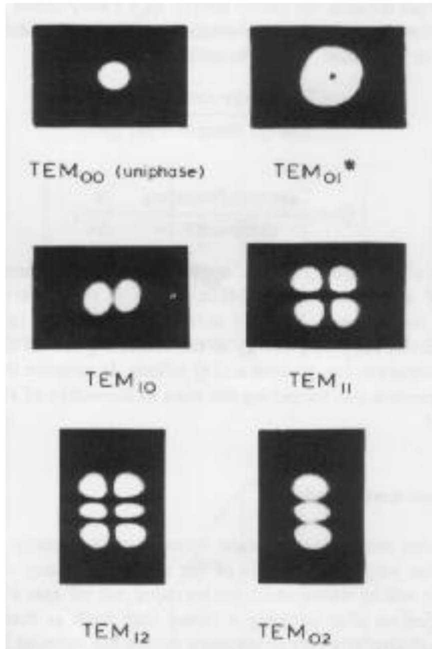


Figure 6.19 Modes

greater the number of modes present within the laser transition linewidth. The number of transverse modes will depend on mirror shape, size, and other aspects of laser construction. When there are a number of modes in the laser output, we refer to the laser as operating multimode.

6.4.3 Single Mode Operation

In many applications including chemical and physical investigation it is desirable to have the greatest possible spectral purity. We can achieve this by operating a cw laser in a single longitudinal and transverse mode. Since an inhomogeneously broadened laser can support several longitudinal and transverse modes simultaneously. Single mode operation can be achieved only by arranging for one mode to have a higher gain than other modes. We can ensure that the cavity will support a single transverse mode only, the TEM_{00} mode, by placing an aperture within the cavity. As the higher order TEM modes spread out more than the TEM_{00} mode an aperture of suitable diameter will transmit the TEM_{00} mode while eliminating the others. All but one of the longitudinal modes can be rejected by reducing the length L of the laser cavity until the frequency separation between the adjacent modes, that is

$\Delta\nu = c/2L$, is greater than the linewidth of the laser transition. Figure 6.17 then shows that the single mode which falls within the transition linewidth is the only one that can oscillate. The disadvantage of this system is that the active length of the laser cavity may become so small as to severely limit the power output. This shortcoming of lower power in single longitudinal mode can be overcome by using Fabry-Perot etalon inside the resonating laser cavity, details of the technique can be found in a number of text books e.g. books given at the end of the Unit.

Problems

6.1 Why plane mirror resonator is called marginally stable?

6.2 Discuss the merits and demerits of unstable resonators versus stable resonators.

6.3 Make a series of sketches showing two mirrors facing each other, with the mirror spacing fixed. Assuming that the centre of curvature C_1 of the right hand mirror is located in the following positions:

(i) C_1 located at the centre of the two mirrors.

(ii) C_1 located to the left of the left-hand mirror.

Indicate by cross hatching those sections of the axis within which the centre of curvature C_2 of the left-hand mirror must be located in order to have a stable resonator.

6.4 An optical cavity of length L is formed by two end mirrors. Set-up the necessary analysis and find out whether the resonator will be stable or unstable, given that

(i) Length of the cavity = $L = R_1 = R_2 = 10$ cm

(ii) $L = 12$ cm, $R_1 = R_2 = 10$ cm

(iii) $L = 12$ cm, $R_1 = 10$ cm, $R_2 = 15$cm

(iv) For all the above parts, draw the stability diagram and show the stability region (if any)

6.5 A resonator is formed by a convex mirror of radius $R_1 = -1$ m and a concave mirror of radius$R_2 = 1.5$ m. What is the maximum possible mirror separation if this is to remain a stable resonator?.

6.6 A resonator is made up of two plane mirrors with a positive lens inserted between the two mirrors. If the focal length of the lens is f and L1 and L2 are the distances of the lens from the two mirrors. Calculate the condition under which the cavity is stable.

6.7 What is the mode separation; how many longitudinal modes could possible if the width of the gain curve is 1.5×10^9 Hz? Take the mirror separation to be 2 m and $\lambda = 588$ nm.

6.8 Consider a confocal resonator of length L = 1 m used for a He-Ne laser at a wavelength, $\lambda = 0.6328$ μm. Calculate the frequency difference between adjacent longitudinal modes.

6.9 Calculate the cavity length for a He-Ne laser, which would sustain only two longitudinal modes. The bandwidth of 632.8 nm He-Ne laser is about 1.5×10^9 Hz.

Books for further reading:

A. E. Seigman, *Lasers*, (University Science Books, California, 1986).

O. Svelto, *Principles of Lasers*, 3rd ed. (Plenum Press, New York, 1989).

P. W. Milonni and J. H. Eberly, *Lasers*, (John Wiley & Sons, New York, 1991).

A. E. Seigman, *Lasers and Masers*, (McGraw-Hill, New York, 1971)

Unit-7

The Laser Output

Objective

In the previous units we have discussed that in general lasers systems have three basic requirements; (i) active medium, (ii) population inversion, and (iii) optical resonator. In this unit we will investigate how these requirements determine the characteristics of the laser output. In the course of this investigation we shall discuss the atomic lineshape associated with a laser transition.

7.1 Introduction

This unit deals with the discussion of the characteristics of the laser beams. Single mode operation of a laser beam makes it useful in many applications where low frequency spread is necessary e.g. high resolution spectroscopy, interferometry. In pulsed lasers, narrowing the pulse duration increases the peak power; this can be achieved by Q-switching. In addition to these, some common properties of laser radiation are discussed. The objective is to make the student familiar with these characteristics of laser beams.

7.2 Lineshape Function

In deriving the expression for small signal gain (unit-4) we assumed that all the atoms in either the upper or lower levels would be able to interact with the perfectly monochromatic beam. In fact this is not so; spectral lines have a finite wavelength (or frequency) spread, that is, they have a spectral width. This can be seen in both emission and absorption. For example, if we measure the absorption as a function of frequency for the transition between the two energy levels E_1 and E_2, we will obtain a bell-shape curve as shown in Figure 7.1a. The emission curve will be the inverse of this Figure 7.1b. The shape of these curves is

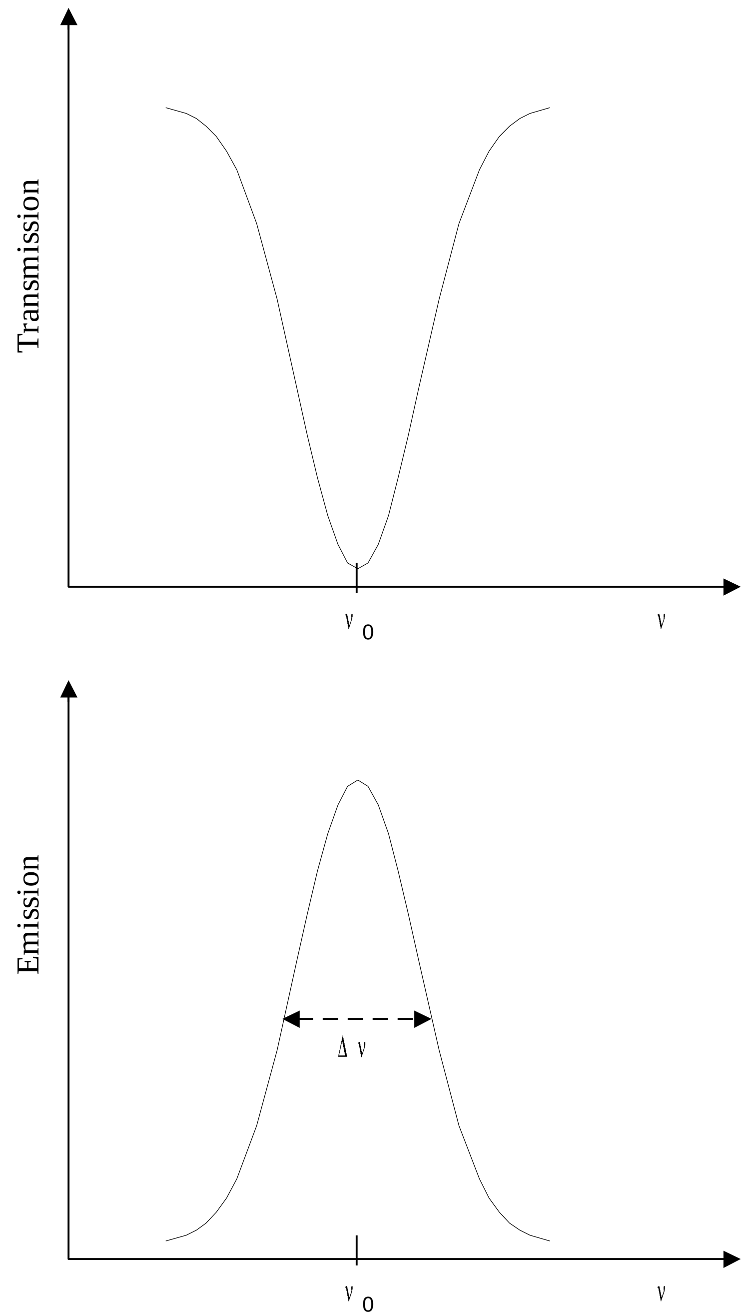


Figure 7.1 (a) The transmission curve for transitions between energy levels E_1 and E_2 and (b) the emission curve for transitions between E_2 and E_1. The precise form of these curves (the lineshape) depends on the spectral broadening mechanisms

described by the lineshape function g(ν). Thus we may define g(ν)dν as the probability that a given transition between the two energy levels will result in emission (or absorption) of a photon whose frequency lies between ν and ν+dν. g(ν) is normalized such that

$$\int_{-\infty}^{\infty} g(\nu).d\nu = 1. \tag{7.1}$$

Therefore, we see that a photon of energy hν may not necessarily stimulate another photon of energy hν. We then take g(ν)dν as the probability that the stimulated photon will have an energy between hν and h(ν+dν).

When a monochromatic beam of frequency ν_s interacts with a group of atoms having a lineshape function g(ν), the small signal gain coefficient may be written as (after equation 4.21):

$$k(\nu_s) = (N_2 - N_1)\frac{B_{21}.h\nu_s.ng(\nu_s)}{c} \tag{7.2}$$

The form of the lineshape function g(ν) depends on the particular mechanism responsible for the spectral broadening in a given transition. The three most important mechanisms are Doppler broadening, collision (or pressure) broadening and natural (or lifetime) broadening. In the subsequent discussions the broadening mechanisms are described briefly.

7.2.1 Natural Broadening

The natural linewidth of an atomic or molecular transition is due to the spontaneous emission from the excited state. The lifetime (τ) and energy spread (ΔE) of the excited state is related by the uncertainty principle (assuming atom or molecule is free from all other interactions):

$$\tau.\Delta E \approx \hbar = \frac{h}{2\pi} \tag{7.3}$$

The corresponding spread in frequency (half width) is:

$$(\Delta\nu)_{nat} = \frac{\Delta E}{h} \approx \frac{1}{2\pi.\tau} \tag{7.4}$$

In natural broadening, the probability that an atom or molecule will emit a photon within this linewidth is same. Therefore, this is also known as homogeneous broadening.

7.2.2 Doppler Broadening

The Doppler effect occurs because of the relative motion of a source and observer. The frequency as measured by the observer increases if the source and observer approach one another and decreases as they move away. This effect applies to a collection of atoms emitting at an optical frequency ν_{12} so that the observed frequency is given by

$$\nu'_{12} = \nu_{12}.\left(1 \pm \frac{v_x}{c}\right), \tag{7.5}$$

where v_x is the velocity of the atom along the direction of observation (we assume $v_x << c$). Since the atoms are in random motion, an observer would measure a range of frequencies depending on the magnitude and direction of v_x. That is, as for the observer is concerned, the collection of atoms would be emitting at a range of different resonant frequencies resulting in a broadening of the emission lineshape. The individual Doppler shifted resonant frequencies contribute to a smooth Doppler broadened lineshape.

The mean squared velocity components v_x depend on the temperature since

$$\tfrac{1}{2} m v_x^2 = \tfrac{1}{2} kT \tag{7.6}$$

where m is the atomic mass, k is the Boltzmann constant, and T is absolute temperature. In thermal equilibrium the gas molecules have Maxwell-Boltzman distribution of velocities and give rise to a line shape function

$$g_{Dopp}(\nu, \nu_o) = \text{Constant}.\exp\left[-\frac{mc^2}{2kT}\left(\frac{\nu - \nu_o}{\nu_o}\right)^2\right] \tag{7.7}$$

The Doppler linewidth (full width of the curve at half the maximum intensity of emission) of the curve is proportional to the square root of the temperature T and is given by

$$\begin{aligned}(\Delta\nu)_{Dopp} &= \frac{2.\nu_o}{c}\left(\frac{2.kT.\ln 2}{M}\right)^{1/2} \\ &= 7.15\text{x}10^{-7}\,\nu_o\sqrt{T/M}\end{aligned} \tag{7.8}$$

where M is atomic weight in atomic mass unit (amu), T is absolute temperature in K, ν_0 and $(\Delta\nu)_{Dopp}$ in Hz. The above equation shows that the Doppler width is small in heavier molecules and can be reduced by some extent at low temperature. Doppler broadening is the predominant mechanism in most gas lasers emitting in the visible. The temperature is elevated, the resonance frequency is high, and the atomic mass is relatively low; all these conditions contribute to Doppler broadening.

Example 7.1: *Calculate the Doppler broadened linewidth for the CO_2 laser transition (λ=10.6μm) and the He-Ne laser transition (λ=632.8nm) assuming a gas discharge temperature of about 400 K. Take the relative atomic masses of carbon, oxygen and neon to be 12, 16 and 20.2 respectively.*

Solution*: The Doppler width for center wavelength λ_0 is given as*

$$(\Delta\nu)_{Dopp} = 7.15\times 10^{-7}.\frac{c}{\lambda_0}\sqrt{T/M}$$

(a) For CO_2 laser: M = 44 amu, λ_0=10.6x10^{-6} m, T=400 K, c=3x10^{8} m/s

$(\Delta\nu)_{Dopp}$ *= 61 MHz*

(b) For He-Ne laser: M = 20.2 amu, λ_0=632.8x10^{-9} m, T=400 K, c=3x10^{8} m/s

$(\Delta\nu)_{Dopp}$ *= 1508 MHz*

The Doppler width of He-Ne laser is much higher than the CO_2 laser.

7.2.3 Collision Broadening

The collision broadening arises from the collisions between molecules. If the collision is considered as abruptly terminating the life in a particular energy level and τ is the mean time between collisions, which ends the life in the state, the linewidth is

$$(\Delta\nu)_p = \frac{1}{2.\pi.\tau} \qquad (7.9)$$

Clearly, the higher the pressure of the gas the more frequently will atoms suffer collisions and the greater will be the spectral broadening.

The Doppler linewidth of molecular lasers such as the CO_2 laser is relatively small because of their low resonant frequencies (in the infrared) and comparatively large molecular masses. In such lasers collision broadening becomes important. Collision broadening also occurs in doped insulator lasers. In these lasers the ions of the active medium may suffer collisions with phonons, which are quantized lattice vibrations.

7.2.4 Homogeneous and Inhomogeneous Broadening

Broadening mechanisms can be classified into homogeneous and inhomogeneous broadening. If all of the atoms of the collection have the same transition center frequency and the same resonance lineshape then the broadening in termed as homogeneous; such as collision and natural broadening. On the other hand, in some situations each atom has a slightly different resonance frequency or lineshape for the same transition. The observed lineshape is then the average of the individual ones, such as Doppler broadening, and the mechanism is termed as inhomogeneous. Local variations of temperature, pressure, and applied magnetic field as well as local variations due to crystal imperfections also lead to inhomogeneous broadening of the emission or absorption lineshapes.

Homogeneous broadening mechanisms leads to a Lorentzian lineshape, which may be written as:

$$g(\nu)_L = \frac{\Delta\nu}{2\pi}\left[(\nu-\nu_0)^2 + \left(\frac{\Delta\nu}{2}\right)^2\right]^{-1} \tag{7.10}$$

where $\Delta\nu$ is the linewidth, that is the separation between the two points on the frequency curve where the function falls to half of its peak value which occurs at frequency ν_0. Putting $\nu=\nu_0$ gives

$$g(\nu)_L = \frac{2}{\pi.\Delta\nu} \tag{7.11}$$

Inhomogeneous broadening mechanisms, on the other hand, lead to a Gaussian frequency distribution, given by:

$$g(\nu)_G = \frac{2}{\Delta\nu}\left(\frac{\ln 2}{\pi}\right)^{\frac{1}{2}} \exp\left[-(\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu/2}\right)^2\right] \tag{7.12}$$

and putting $\nu=\nu_0$ gives

$$g(\nu)_G = \frac{2}{\Delta\nu}\left(\frac{\ln 2}{\pi}\right)^{\frac{1}{2}} \tag{7.13}$$

Because of these various broadening mechanisms we can no longer treat a group of atoms as though they all radiate at the same frequency. Instead, we must consider a small spread of frequencies about some central value. It might then be expected that the output of the laser would contain the same distribution of frequencies as the broadened transitions of the atoms in the medium. This is; in fact, not the case of as the spectral character of the laser output is different from that of spontaneous emission in the same medium. Two factors account for this difference: namely, the effects of optical resonator and the effect of the amplification process on the irradiance.

7.3 Q-Switching

Q-switching is a way of obtaining short, powerful pulses of laser radiation. Here Q is the quality factor of the laser resonator and the term Q-switching refers to an abrupt change in the cavity loss. Specifically, it is a sudden switching of the cavity Q from a low value to a high value, i.e., a sudden lowering of the cavity loss. A high–Q cavity is one with low loss, whereas a lossy cavity will have a low Q.

Single high power pulses can be obtained by introducing time or intensity-dependent losses into the cavity. If there is initially a very high loss in the laser cavity, the gain due to population inversion can reach a very high value without laser oscillations occurring. The high loss prevents laser action while energy is being pumped into the excited state of the medium. When a large population inversion has been achieved, the cavity loss is suddenly reduced and laser oscillation will suddenly commence. On Q-switching, the threshold gain decreases immediately while the actual gain remains high because of the large population inversion. Due to the large difference between the actual and threshold gain the laser oscillations within the cavity build up very rapidly and all of the available energy emitted in a single, large pulse. This quickly depopulates the upper lasing level to such an extent that the gain is reduced below threshold and the lasing action stops. The time variation of some parameters during Q-switching is shown schematically in Figure 7.2. Q-switching dramatically increases the peak power obtainable from lasers. When the laser is Q-switched, the result is a single spike of great power, typically in the megawatt range, with a duration of a few nanoseconds. It is important to note that, although there is a vast increase in the peak power of a Q-switched laser, the total energy emitted is less than in non-Q-switched operation due to losses associated with the Q-switching mechanism.

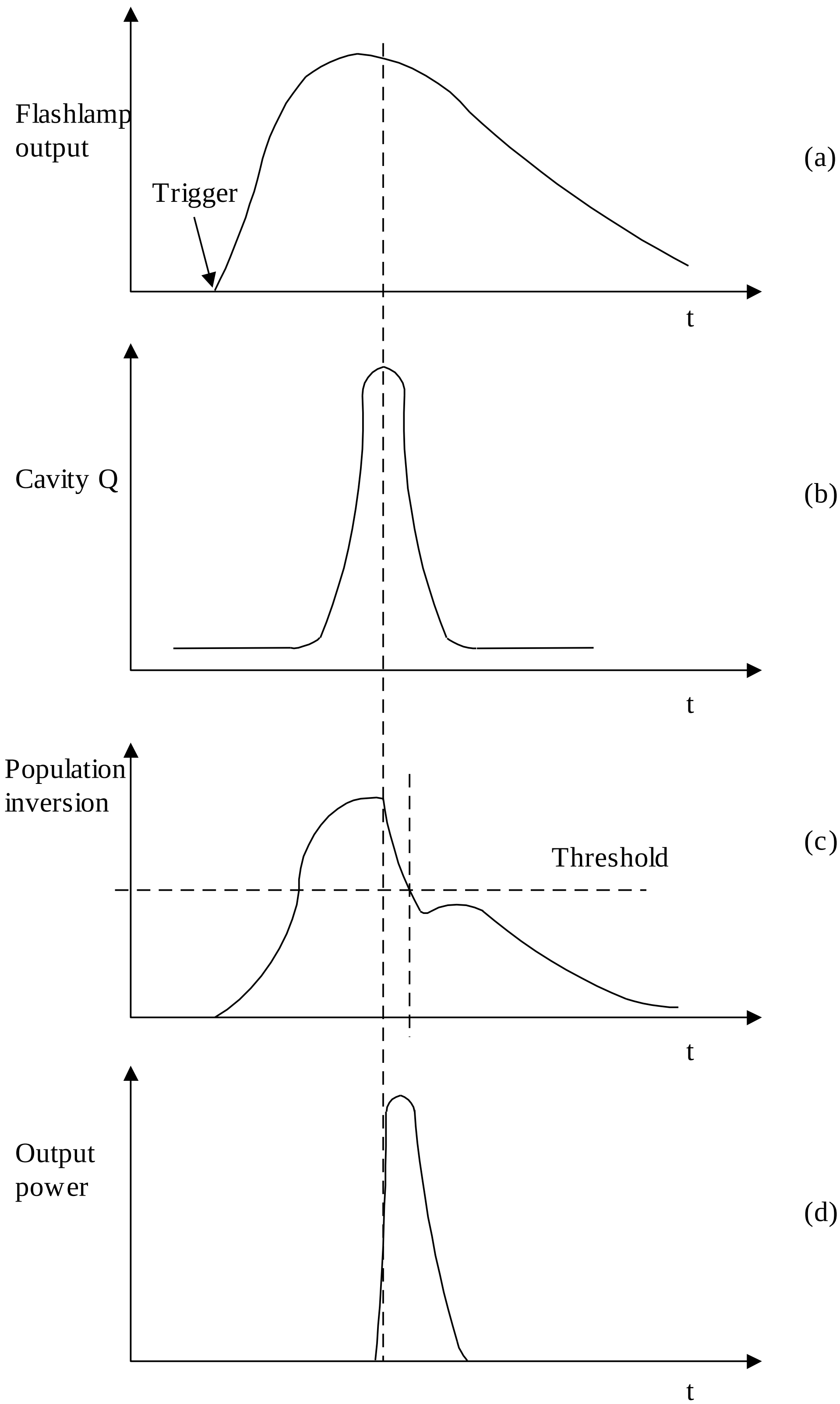


Figure 7.2 Schamatic representation of the variation of the parameters flashlamp output, (b) cavity Q, (c) population inversion and (d) output power a function of time during the formation of a Q-switched laser pulse.

Q-switching is carried out by placing a closed shutter, that is the Q-switch within the cavity, effectively isolating the cavity from the laser medium. After the laser has been pumped the shutter is opened so restoring the Q of the cavity. There are two important requirements for effective Q-switching. These are:

(i) the rate of pumping must be faster than the spontaneous decay rate of the upper lasing level otherwise the upper level will empty more quickly than it can be filled, so that a sufficiently large population inversion will not be achieved; and

(ii) the Q-switch must switch rapidly in comparison to the build up of the laser oscillations otherwise the latter will build up gradually and a longer pulse will be obtained so reducing the peak power. In practice the Q-switch should operate in a time less than 1 ns.

There are different methods of achieving Q-switching.

7.3.1 Rotating Mirror Q-Switch

This is the first method used for Q-switching of solid state laser. This method involves rotating one of the mirrors at very high angular velocity (Figure 7.3) so that optical losses are high except for the brief interval in each rotation cycle when the mirrors are parallel. Just before this point is reached a trigger mechanism initiates the flashlamp discharge to pump the laser. As the mirrors are not yet parallel the population inversion can build up without lasing. When the mirrors become parallel, Q-switching occurs allowing the Q-switched pulse to develop as illustrated in the Figure 7.2.

Although rotating mirror type Q-switch is cheap, reliable and rugged the method suffers from the major disadvantage of being slow. This results in an efficient production of Q-switched pulses with low peak power than can be produced by other methods.

7.3.2 Electro-Optic Q-Switch

Some crystals (and liquids) have the ability to rotate the plane of polarization of light passing through them, they are called optically active. Similar effect can be seen in some crystals by applying the electric field across them, the phenomenon is known as electro-optic effect. The

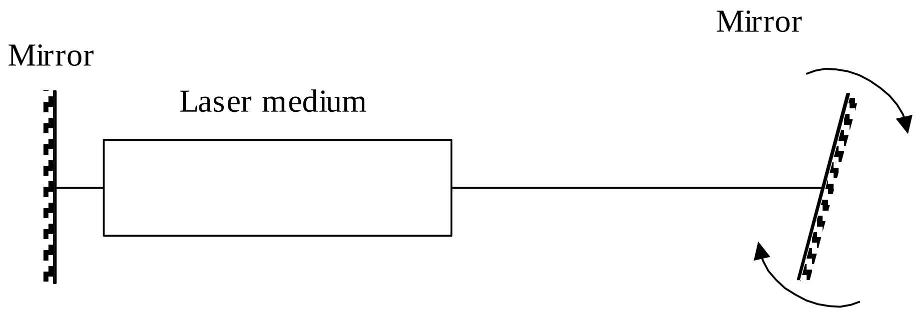


Figure 7.3 A laser cavity with a rotating mirror for Q switching.

electro-optic modulator can be used as fast Q-switch. When a Pockels cell is used and the laser output is not polarized then a polarizer must be placed in the cavity along with the electro-optic cell as shown in Figure 7.4.

A voltage is applied to the cell to produce a quarter wave-plate, which converts the linearly polarized light incident on it into circularly polarized light. The laser mirror reflects this light and in so doing reverses its direction of rotation. On passing again through the electro-optic cell it emerges as plane polarized light, but at 90^o to its original direction of polarization. The polarizer does therefore not transmit this light and the cavity is ‘switched off’. When the voltage is reduced to zero, there is no rotation of the plane of polarization and Q-switching occurs. The change of voltage, which is synchronized with the pumping mechanism, can be accomplished in less than 10 ns and very effective Q-switching occurs.

7.3.3 Passive Q-Switch

The electronic circuits can be avoided in Q-switching of lasers using absorption characteristics of certain dyes. This passive Q-switching can be obtained by placing in the laser cavity a material that exhibits an absorptivity that decreases with increasing irradiance, as shown in Figure 7.5. An example of such a material is a saturable dye that possesses an absorption band at the lasing transition. At the beginning of the excitation flash, the dye is opaque due to the large number of unexcited molecules that can absorb the light. As in the other Q-switching mechanism, the low cavity Q prevents lasing action and allows a larger population inversion to be achieved than would otherwise occur. As the light irradiance in the cavity increases, more excited states of the dye are populated, until all possible excited

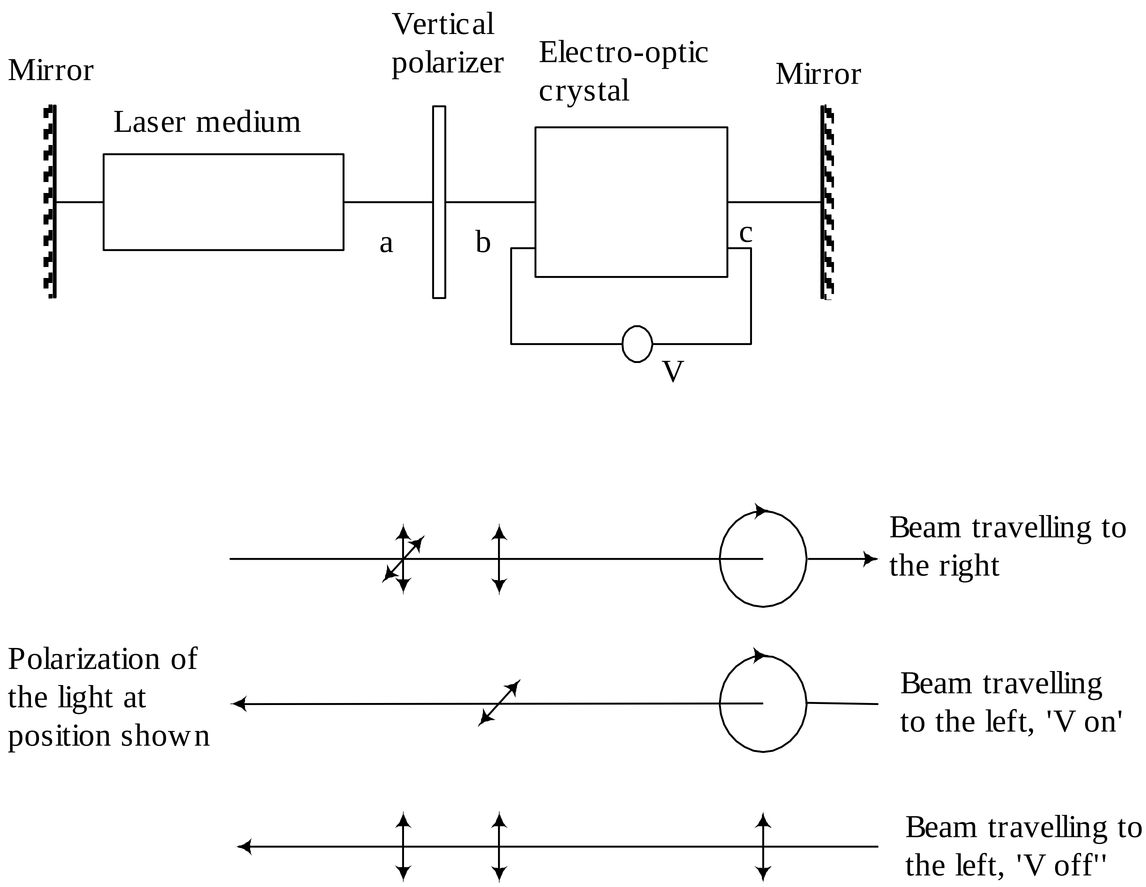


Figure 7.4 Electro-optic crystal used as Q-switch. With the voltage V on, the electro-o crystal act as a quarter wave plate and converts the vertically polarized light at b i circularly polarized light at c. The reflected light is converted to horizontally polariz light and eliminated by the polarizer so that cavity Q is low. With V off, the crysta ineffictive and the cavity Q is high.

states of the dye are filled. At this point, the dye can no longer absorb at the laser wavelength and is said to be *bleached*. The abrupt reduction in cavity losses causes Q-switching to occur. Passive Q-switching has the great advantage of being extremely simple to implement. The only equipment necessary is a small dye cell (which is also available in the form of a thin dye sheet) inserted into the cavity between the lasing medium and one of the ends mirrors.

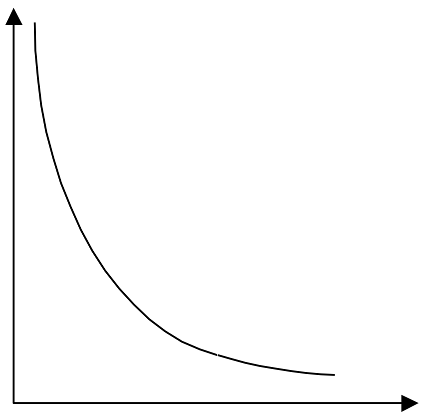


Figure 7.5 Absorption as a function of incidence light irradiance for a satura absorber.

7.4 Laser Beam Properties

The output beam from a laser system is basically consists of electromagnetic radiation, or light. There are some important and fundamental differences of laser light and light emitted from any other source of electromagnetic radiation. Laser beams are often described as being different from ordinary light sources in coherence, spectral purity, divergence, etc. These phrases refer to some characteristics of laser beams that we will review briefly in this section.

7.4.1 Monochromaticity

The word monochromatic is derived from Greek language, meaning single color. In the scientific world it is used for electromagnetic waves of single frequency. In fact no light source, including laser, is capable of producing absolutely monochromatic light, we can only make better and better approximations to the ideal. We might begin with a white light source that produces light of all colors of the spectrum, and filter it with a piece of colored glass. The monochromaticity of the filtered light is now as good as the filter. If the radiation from a gas discharge source, such as neon sign or a sodium vapor lamp, is directed through a prism, a series of lines of different colors is seen on a screen.

The degree of monochromaticity of light from some sources can quantitatively be determined by characterizing the spread in frequency of a line by $\Delta\nu$, the linewidth of the source. This spread can also be represented in term of wavelength, i.e., $\Delta\lambda$. The two spread are related as

$\Delta\nu = -(c/\lambda^2).\Delta\lambda$. This frequency spread depends upon the light source and the level of excitation it can range from the broad, $\Delta\lambda \sim 300$ nm, (in the case of white light source) to the narrow, $\Delta\lambda \sim 0.01$ nm (for gas discharge lines). By isolating one of these narrow lines with a suitable filter, we can achieve monochromaticity as good as the width of a single emission line.

If the gas in the above discharge can be made to undergo laser action, this particular emission line is replaced by a series of even narrower lines, which represent different modes. By suppressing all but one of these modes, further monochromatic source of light can be obtained. But even this exceedingly narrow line contains a small spread of different frequencies. If the light consisted of radiation oscillating at a single frequency, the line would be infinitely narrow ($\Delta\nu = 0$) and the light would be absolutely monochromatic. Absolute monochromaticity is an unattainable goal that can only approach by refining our light sources.

The short term spectral purity of a single mode laser can range from a few tens of MHz down to only a few Hz in a highly stabilized system. In fact, it is the laser cavity and not the laser transition that is primarily responsible for these spectral properties. The short term frequency jitters and the long term frequency drift of laser oscillator usually result primarily from mechanical vibrations and noise, thermal expansion, and other effects that tend to change the length L of the laser cavity. Very highly stabilized laser oscillations can nonetheless have long term absolute frequency stability better than 1 part in 10^{10}, and short term spectral purity as high as 1 part in 10^{13}, making them equal to or better than the best atomic clocks available in any frequency range. The ultimate limit on laser spectral purity is finally set by quantum noise fluctuations caused by the spontaneous emission from the atoms inside the laser cavity.

7.4.2 Coherence

Coherence probably is the best-known property of laser light. Light waves are coherent if they are in phase with each other, i.e., if their peaks and valleys are lined up at the same point, as shown in Figure 7.6. Two things are necessary for light waves to be coherent. First, the light waves must start out having the same phase at the same position. Second, their wavelengths must be the same, or they will drift out of phase because the peaks of the peaks of the longer wave.

The laser light is coherent because stimulated emission is coherent with the light wave and photons that stimulates it. The stimulated wave has the same phase and wavelength. It, in turn, can stimulate the emission of other photons, which are in phase and have the same wavelength both with it and the original wave.

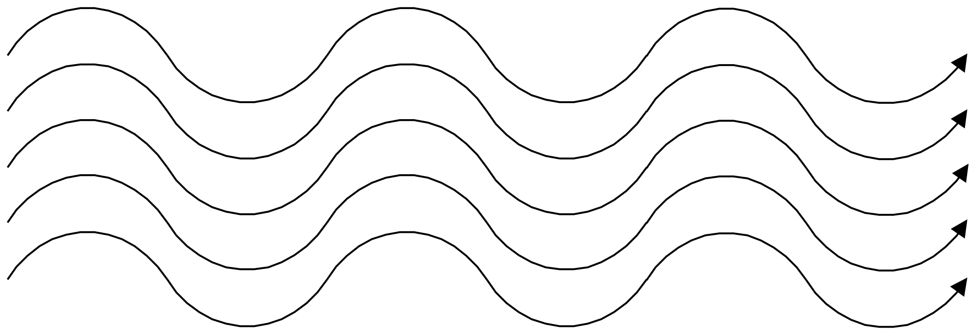


Figure 7.6 Coherence of laser light.

The preceding statement gives the impression that all laser light is perfectly in phase, but this is not the case. Not all photons in a laser beam are descended from the same original photon by stimulated emission, so they don't start out in phase. Even if they were, the uncertainty principle leads to tiny variations in the wavelengths of the emitted light, which accumulate and become significant after the light goes a long distance. In addition, tiny fluctuations within the laser, such as thermal gradients or vibrations, can affect one light wave in a different way than others, degrading coherence. Thus, there is no such thing as "perfect" coherence.

Not all laser light is equally coherent. In fact, the light from some laser is almost incoherent. The reasons lie inside the lasers themselves. Some lasers emit a broader range of wavelengths than others do, and we saw earlier, light waves must have the same wavelength to be coherent. Thus a laser should oscillate in only one frequency, more than one frequencies oscillations reduce the degree of coherence. Lasers with high gain tend to emit a broad range of wavelengths because a range of wavelengths (generated by spontaneous emission) can stimulate emission. In other words, high gain lasers are likely to amplify many different spontaneously emitted light waves. Because of the uncertainty principle, the range of wavelengths in a pulse increase as the length of a pulse decreases, so the shortest pulses tend to have the broadest wavelength ranges. On the other hand, continuous-wave lasers can have the narrowest wavelength range. Add this all together and one finds that the most coherent beams come from continuous-wave, low-gain lasers operating in the low order mode.

If we look closely, there actually are two kinds of coherence: temporal and spatial. Temporal coherence measure how long light waves remain in phase as they travel (the term “temporal” comes from the degree of coherence is compared at different times). Light wave become incoherent as differences in their paths or wavelengths make them drift out of phase. All light has some temporal coherence, but only over a characteristic “coherence length,” which is close to zero for ordinary light bulbs but can be many meters for lasers. In fact the degree of monochromaticity is related to the coherence length by:

$$\text{Coherence length, } L = \frac{c}{\Delta\nu}$$

where $\Delta\nu$ is the linewidth of the source radiation. The coherence time of the radiation is related to $\Delta\nu$ through the relation

$$\tau_c = \frac{1}{\Delta\nu}$$

Example 7.2: *Find coherence length for the following:*

(a) Light bulbs emit light from visible to infrared, i.e., 400 nm to 1000 nm

(b) An ordinary semiconductor laser operating at 800 nm with wavelength range of 1 nm.

(c) An ordinary helium neon laser with frequency bandwidth of 1500 MHz.

(d) A frequency stabilised helium neon laser with frequency bandwidth of 1 MHz.

Solution: *The coherence length* $L = c/\Delta\nu$, *or in the units of wavelengths* $L = \lambda^2/\Delta\lambda$

(a) Average emission wavelength from a bulb is 700 nm, and wavelength spread $\Delta\lambda$*=600 nm, therefore,* $L = 8.2\times10^{-7}$ *m.*

Hence ordinary bulb has a very short coherence length of 0.82 micrometer.

(b) Since λ=800 nm, and $\Delta\lambda$=1 nm, therefore, coherence length L = $6.4x10^{-4}$ m

The coherence length of an ordinary semiconductor laser is also not very large and is only 0.64 mm.

(c) Helium neon laser with a frequency bandwidth, $\Delta\nu$ = 1500 MHz, the coherence length, L, becomes 0.2 m.

The ordinary helium neon laser has a coherence length of 20 cm that is sufficient for some interferometric applications and holography.

(d) A frequency stabilised helium neon laser with frequency bandwidth, $\Delta\nu$=1MHz, L= 300 m.

which is sufficiently large. It has applications in interferometry, high-resolution spectroscopy, etc.

Spatial coherence, on the other hand, measures the area over which light is coherent. If a laser emits a single transverse mode (i.e. laser beam has Gaussian intensity profile in the transverse direction), its emission is spatially coherent across the diameter of the beam, at least over reasonable propagation distances.

7.5.3 Directionality

We think of laser beams as tightly focused and straight, but once they go for enough from the laser, they actually spread out slightly with distance. The spreading is called beam divergence. Laser beams, in general, have very low divergence. The light in a laser is contained between two highly reflective mirrors. As a first approximation, one can think these mirrors as collimating apertures. The mirrors have such a high reflectivity that a wave is reflected many times with only small portion of it being transmitted by the mirrors. The multiple reflections increase the distance the light travels, while confining it within a very small region between the mirrors. Thus, because of the long distance travelled, the curvature of the waves is very small and the light waves emerging from the laser are nearly planar. If we consider a single point source located between the two mirrors, we can get some idea of the collimating property of multiple reflections using only geometry. Each time a light wave reflects off a mirror, the distance between it and the point source that generated it increases. Since the output mirror transmits a smaller region of that wave front after each reflection, it serves as a collimating aperture for the wave. This collimating property can be visualised on a barber or beautician's chair between mirrors on opposite walls. In many lasers the number of reflections is over 50 and as high as several hundred (this number can be calculated from the mirror transmittance). The beam transmitted from the tube looks as though it comes from a point some 100 cavity lengths distant with 100 apertures collimating the beam.

The low divergence of the laser beam implies that the energy carried by the laser beam can be collected easily and focused into a small area. For conventional sources, where the radiation spreads out into a solid angle of 4π steradian, efficient collection is almost impossible. While for laser beam divergence angle is so small that efficient collection is possible even at large distances from the laser.

A single transverse-mode laser can produce an output beam that has almost uniform amplitude and phase across its full output aperture of diameter d. Such a beam can propagate for a sizeable distance with very little spread. This beam has a very small far-field angle at large distances and can be focused into a spot only a few wavelengths in diameter.

The directionality of the laser beam is expressed in terms of the full angle beam divergence, which is twice the angle that the outer edge of the beam makes with the center of the beam (Figure 7.7). The divergence tells us how rapidly the beam spreads when it is emitted from the laser. Although the divergence angle can be given in fractions of degrees, minutes, or seconds, it is common to specify the beam divergence in radians, where 2π radians equal 360^o. For a typical helium neon laser, the beam divergence is about 1 milliradian (10^{-3} radians). Using the small angle approximation ($\tan\theta \sim \theta$) one can easily show that this typical laser beam increases in size about 1 mm for every meter of beam travel.

Example 7.3: *Calculate diameter of the helium neon laser with divergence of 1 mrad at a distance of 2 km.*

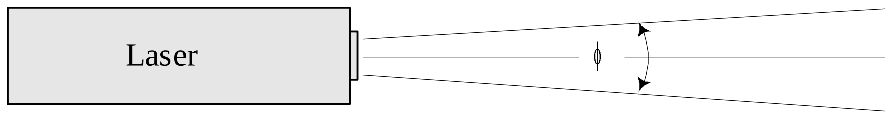


Figure 7.7 Full angle beam divergence of a laser is twice the angle of a laser be
and the beam center.

Solution: *The small angle approximation is valid for such a small angle, i.e., tanθ~θ, we can find the diameter of the laser beam at 2 km as:*

Diameter, d = distance x divergence angle

d = 2000x0.001=2 meters.

Therefore the beam diameter of helium neon laser at 2 km is 2 meter for beam divergence of 1 mrad.

7.4.4 Focusing Properties of Laser Radiation

The minimum spot size to which a laser beam can be focused is determined by diffraction. A single mode beam can be focused into a spot, which has dimensions of the order of the wavelength of light. The imperfections in the optical system may mean that we cannot achieve this in practice. A useful relation for estimating the spot size is that the radius r, at the focal plane of a lens of focal length f is given by

$$r = f.\theta \tag{7.14}$$

where θ is the beam divergence angle in radians. Diffraction provides the lower limit to divergence of a laser beam. In this limit θ is given approximately by λ/D, where D is the limiting aperture diameter, therefore, we have

$$r \approx f\,\frac{\lambda}{D} \approx \lambda.F \tag{7.15}$$

where F is the F-number of the lens. It is impractical to work with F-numbers much smaller than unity, so that r is of the order of λ.

Thus, for example, if we have a 10 mW He-Ne laser with a beam divergence of 0.1 mrad, then an F:1 lens will produce a focused spot with an area of about 10^{-12} m^2 and the power per unit area near the center of the spot will be around 10^{10} W m^{-2}.

If the beam divergence is large the power density is reduced. The insulating crystal lasers generates very high peak powers, they can easily produce high irradiance. A focal area of 10^{-7} m^2 is typical for such lasers giving rise to typical average irradiance of 10^5 W m^{-2} and peak irradiance of 10^8 Wm^{-2}.

Such high irradiance leads to the use of lasers in the drilling, cutting, welding and heat treatment of large number of different materials. The focussing properties of laser radiation are also important for low power applications, e.g., preparation and readout of some home videodisc systems. The information is imprinted on the videodisc in digital form by forming small pits in the surface of the disc with laser. A low power laser to provide a video signal for playback on a television set subsequently reads these pits.

7.5.5 Brightness

The primary characteristic of laser radiation is that lasers have a higher brightness than any other light sources. We define brightness as power emitted per unit area per unit solid angle. The relevant solid angle is that defined by the cone into which the beam spreads. Hence, as lasers can produce high levels of power in well-collimated beams, they represent sources of great brightness.

The brightness is affected by the presence of additional modes, for often, as laser power is increased, the number of mode increases but the brightness remains almost constant. Typical values of brightness from different sources are:

Sun	~ 1.3×10^6 W m^{-2} sr^{-1}
He-Ne laser	~10^{10} W m^{-2} sr^{-1}
Q-switched ruby laser	~10^{16} W m^{-2} sr^{-1}
High power Nd:glass laser	~10^{21} W m^{-2} sr^{-1}

High brightness is essential for a large number of applications, including material processing, medical application, defense application, fusion, etc.

Problems

7.1 Reduction in linewidth increases the coherence length of the laser beam. Give physical reasons for this fact.

7.2 What are the merits and demerits of Q-switching in a laser system? Give a comparison of different Q-switching techniques.

7.3 A certain laser has frequency bandwidth of 1500 MHz. Represent it in terms of wavelength range if (i) λ = 632.8 nm; (ii) λ = 3.39 μm.

7.4 A coherence length of 50 km is desired for a laser beam. What should be maximum wavelength range for it.

7.5 Calculate the divergence angle of laser beam, given that its radius becomes 2 meters at the distance of 4 km.

7.6 The half-width of the 10.6 μm transition of a low-pressure CO_2 laser is 60 MHz, calculate the coherence length of the laser. If the cavity length is 1 meter show that not more than one mode will oscillate.

Books for further reading:

D. C. O'shea, W. R. Callen, and W. T. Rhode, *An introduction to lasers and their applications*, (Addison-Wesely, California, 1978).

A. E. Seigman, *Lasers*, (University Science Books, California, 1986).

O. Svelto, *Principles of Lasers*, 3rd ed. (Plenum Press, New York, 1989).

P. W. Milonni and J. H. Eberly, *Lasers*, (John Wiley & Sons, New York, 1991).

A. E. Seigman, *Lasers and Masers*, (1971).

Unit-8

Laser Systems

Objective

In this unit some typical laser systems are described. After studying the basic principles of laser, understanding of a few represented systems may clarify the ideas developed in the previous units.

8.1 Classes of Lasers

The common laser systems can be classified into four groups: gas, liquid, solid-state and semiconductor lasers. Before discussing laser systems, it might be useful to remind some of the basic requirements which must be satisfied for laser operation.

Firstly, there must be an active medium, which emits radiation in the required region of the electromagnetic spectrum. Secondly, a population inversion must be created within the medium; this requires the existence of suitable energy levels associated with the lasing transition for pumping.

Thirdly, for true laser oscillation there must be optical feedback at the ends of the medium to form a resonant cavity. The first two conditions can provide light amplification but not the highly collimated, monochromatic beam of light which makes lasers useful.

In the following, we discuss different types of lasers.

8.2 Gas Lasers

Gas lasers are most widely used type of lasers; they range from the low power helium-neon laser (commonly found in teaching laboratories) to the very high power carbon dioxide lasers which have many industrial applications. Basically there are three different classes of gas lasers according to whether the transitions are between the electronic energy levels of atoms or ions, or between vibrational/rotational levels of molecules. These types are called neutral atom gas lasers, ion gas lasers and molecular gas lasers. In general, the energy involved in the lasing processes is well defined and an absence of broad bands effectively eliminates the possibility of optical pumping. Though other methods can be used, most gas lasers are excited by electron collisions in a gas discharge.

Various types of gas lasers are available commercially, e.g. Helium-Neon Laser, Argon and Krypton Lasers, Copper Vapor Laser, Nitrogen Laser, Carbon Dioxide Laser, Excimer Laser etc. In order to have the knowledge about the inside picture of a gas laser, we will discuss the most common type of gas lasers, i.e., the He-Ne laser.

8.2.1 Helium Neon Laser

Helium-neon laser is commonly used neutral atom laser, because it is compact, portable, the low divergent, high coherence length, easily usable, and relatively inexpensive source of laser light. Laser action is obtained from the transitions of the neon atoms (active medium), while helium is added to the gas mixture to greatly enhance the efficiency of the pumping process. Although it has been found that the neon gas alone can provide laser action but the output is enhanced about 200 times when helium is mixed in neon in proportions of 85% helium and 15% neon.

The helium-neon laser oscillates on many wavelengths, which include 632.8 nm (red), 543 nm (green) and the infrared at 1.15 μm & 3.39 μm and many other wavelengths. The He-Ne laser oscillating on λ=1.15 μm transitions was the first gas laser to be operated and also gave the first demonstration of continuous wave (cw) operation in the laser world. But λ = 632.8 nm (red) has become the most popular due to its visibility to human eye.

The output of He-Ne laser is continuous and upto 10 mw(red) are commonly available. A few special-purpose Helium-Neon lasers, relatively larger in size, can produce upto 60 mw of red light.

He-Ne lasers can be used for many applications where a low power is needed e.g. for character reading, meteorology, holography, videodisk memories. They have many uses for educational, demonstration and alignment purposes.

8.2.1.1 Pumping Mechanism

The He-Ne laser is pumped electrically. A longitudinal DC electrical discharge is maintained in the narrow tube containing the gas at a pressure of about 10 torr. In order to provide the current through the tube, one must first provide a sufficiently high voltage to break down the gases in the tube. In a typical small helium-neon laser, the breakdown voltage may be around 3400V. After the breakdown voltage is applied, a plasma is created in the tube. This results in a large increase of current. After the breakdown, the voltage required to maintain the discharge with currents of the order of 10 to 20 milli amperes, is considerably reduced. A typical operating voltage might be around 1350 volts. In order to restrict the current flow to a safe value, a resistor is placed in series with the laser tube.

The pumping process can be described as follows. The first step is the excitation of helium atoms by electron collision to one of the two metastable states designated 2^1S and 2^3S; this is represented by

$$e_1 + He = He^* + e_2$$

where e_1 and e_2 are the electron energies before and after the collision. While in one of the excited states (He*), the helium atoms can transfer their energy to ground state neon atoms with which they may collide. The probability of this resonant transfer of energy is proportional to $\exp(-\Delta E/kT)$ where ΔE is the energy difference between the excited states of the two atoms involved, k is Boltzmann's constant and T is the absolute temperature. The energy level diagram for helium and neon is shown in Figure 8.1. This diagram shows that there is a group of neon levels at almost the same energies as each of the two excited helium states and resonant transfer thus occurs quite readily. The energy transfer is represented by

$$He^* + Ne = Ne^* + He$$

A population inversion is thus created between the 3s and 2p, between 3s and 3p, and between 2s and 2p states of neon. Transitions between the 3s and 2s levels and 3p and 2p

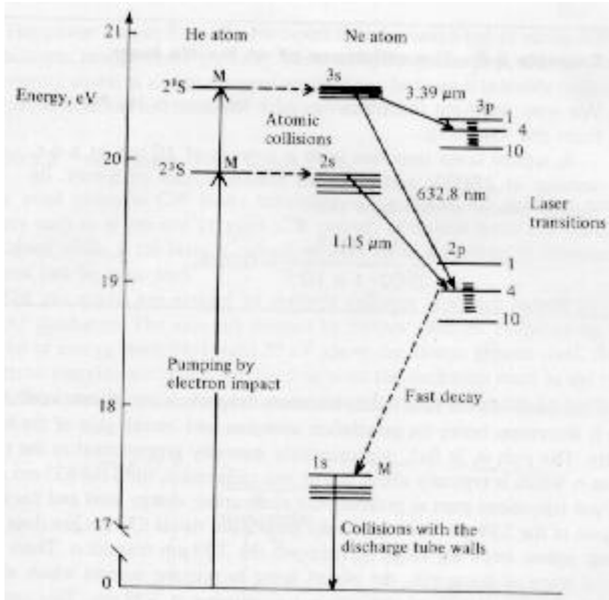


Figure 8.1 Energy levels relevant to the operation of He-Ne laser. M indicates a metastable state.

levels are forbidden by quantum mechanical selection rules. In addition, the decay time of the s states ($\tau_S \cong$ 100 ns) is an order of magnitude longer than the decay time of the p states ($\tau_p \cong$ 10 ns). The p states are rapidly depopulated by spontaneous emission to lower levels. From the ground state they can be again excited to the upper laser level by energy transfer from helium atoms. These parameters satisfy the conditions for operation as a cw laser. The 3s $\rightarrow$ 2p, 2s $\rightarrow$ 2p, and 3s $\rightarrow$ 3p transitions are optically allowed and hence can give rise to various laser wavelengths including 632.8 nm (red), 543 nm (green), 1.523 μm (IR) 3.39 μm (IR), respectively as shown in Figure 8.1.

8.2.1.2 Suppression of the Oscillations other than 632.8 nm

Helium-neon laser can oscillate on many wavelengths, but red laser (λ = 632.8 nm) is widely used. Because of the desirability of visible operation, most He-Ne lasers today are constructed to operate at λ = 632.8 nm (red). We see from the energy level diagram for neon that there is a strong competition between the 0.6328 μm line and the other lines.

When the 1.15 μm line is lasing, it tends to fill the lower laser level for the 0.6328 μm line which will effect the population inversion for 0.6328 μm. Infrared radiation of 3.39 μm wavelength frequently accompanies the emission of the visible line. This transition originates at the $3s_2$ level, the starting level of the visible radiation. So the operation of 3.39 μm laser depletes the $3s_2$ level and hence effects the population inversion required for the visible laser, i.e. this transition uses up atoms that would otherwise contribute to the 0.6328 μm line.

Thus in many practical cases one tries to suppress oscillations of the infrared lines. There are several ways of doing this; the easiest one is to prepare mirrors in such a way that they have high reflectivities at 633 nm and high transmission at 3.39 μm. This can be achieved quite easily by using the multi-layer coated mirrors, which have a wavelength-dependent reflectance. The very low absorption loss of such mirrors is an essential feature as the gain in the He-Ne medium is quite small.

8.2.1.3 Selection of Different Visible Wavelengths

He-Ne laser not only emits 632.8 nm in the visible but it also emits other visible wavelengths, which have very low gain as compared to 632.8 nm. So a He-Ne laser may be operated in the visible region other than 632.8 nm. It can be done by insertion of a prism in the laser cavity. The operation proceeds from the same upper level as the 0.6328 μm line but to different lower sublevels. The dispersion of the prism causes the different wavelengths passing through it to travel in slightly different directions. The prism is rotated so that only one of these wavelengths will strike the mirror normally and be reflected directly back. In this way any one of the several transmission can be favored over the others. Recently laser manufactures have begun offering the He-Ne laser that emit at weaker visible lines particularly the 543 nm (green light).

8.2.1.4 Output Power versus Current

The output power as a function of tube current is shown in Figure 8.2. For low tube currents, the gas discharge is unstable. The plasma flickers on and off and there is no output. For current greater than few milli amperes, a stable plasma is produced and the laser output increases with increasing current. At still higher tube currents, the output power saturates and begins to decrease. This is probably the result of heating of the gas and a decrease in the efficiency of excitation at increased current. In addition, operation at excessive values of currents may decrease the tube life. Thus it is essential to add the ballast resistor in order to keep the tube current within the desired limit.

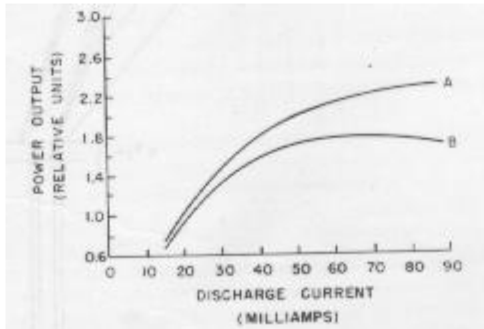
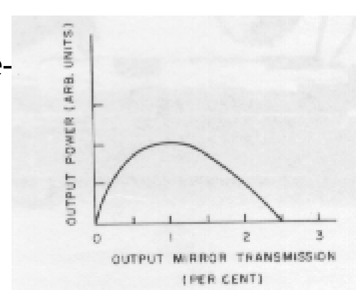


Figure 8.2 Output of He-Ne laser as a function of discharge current. Curve A is for a gas mixture of 0.5 torr of ^{3}He and 0.1 torr of Ne. Curve B is for 0.5 torr of He and 0.1 torr of Ne.

8.2.1.5 Output Power versus Output Mirror Transmission

The gain of the He-Ne laser at 0.6328 μm is relatively low, so that only very small losses can be tolerated in the laser cavity. Thus the mirrors must be of high quality with low scattering losses. The rear cavity mirror is totally reflective. The output mirror typically has transmission around 1 or 2 %. If the mirrors are external to the gas-filled laser tube, Brewster angle windows are necessary on the tube. The output of the helium-neon laser as a function of mirror transmission is shown in Figure 8.3. At zero transmission, the output is of course zero. With increasing transmission, the output increases upto a maximum value. Above the maximum value, the total losses in the laser cavity including the loss due to the output, become too great and the output power decreases. At values of the transmission above 2.5 %, the total losses exceed the gain which in helium neon laser is small, so that the laser operation ceases. Because of the small gain in helium-neon laser, the optimum output coupling is low, less than 1%. It, therefore, follows that the intracavity beam is upto 200 times more intense than the output beam.

Figure 8.3 Output of He- rror transmission.



8.2.1.6 Mirror Arrangements

The mirrors forming the resonant cavity are sometimes cemented to the ends of the discharge tube. Alternatively the mirrors can be external to the tube which is then sealed with glass windows, which are orientated at the Brewster angle to the axis of the tube. This arrangement allows 100% transmission for the radiation with its electric vector vibrating parallel to the plane of incidence. The arrangement ensures the maximum possible gain (minimum losses) in each round trip. Due to the Brewster windows, output is plane polarized. Although this arrangement is slightly more complicated than the earlier one, but it enables us to insert frequency stabilizing, mode selecting and other devices into the cavity. The mirrors can also be changed to allow operation with other output characteristics and other wavelengths.

8.2.1.7 Gain Diameter Relationship

In the helium-neon laser, the gain is an inverse function of the diameter of the gas tube. This probably occurs because the lower level of the laser transition is depopulated by the collisions of the neon atoms with the walls of the tube. Thus to maintain the population inversion and to keep the laser operation, the neon atoms must be able to collide readily with the walls of the tube. In order that the collision rate of the neon atoms with the walls be high, the diameter of the tube must be small. But it limits the helium-neon laser output power because the volume of the gas can't be increased beyond a certain limit for smaller diameter of the tube.

8.2.1.8 Longitudinal Modes

In the gas laser tube, due to the rapid motion of the atoms or molecules, laser transition does not consist of a sharp line. The Doppler effect causes it to be broadened to a smooth Gaussian profile. The typical Doppler broadened linewidth (FWHM) of a red helium neon laser is 1.5 GHz which corresponds to coherence length of 20 cm. Superimposed on the Doppler broadened, gain curve is the resonant cavity function, with the mode spacing given by:

$$\Delta\nu = c/2L, \tag{8.1}$$

where c is the velocity of light and L is the length of the cavity.

For a 0.5 m long cavity, the mode spacing is 300 MHz. For such length of a cavity, laser output therefore consists of four to five longitudinal modes separated by 300 MHz. The sharpness of these modes depends on the homogeneous broadened mechanisms and is typically 1 MHz. The relative intensities of the cavity modes is defined by the Doppler broadened gain curve.

If one needs a longer coherence length or a narrow bandwidth, the length of the helium neon laser cavity is adjusted such that the output will exhibit either one mode or two, depending on the precise length of the cavity. In fact this implies $L < 10$ cm.

8.2.1.9 Transverse Modes

Most commercial helium-neon lasers are now constructed so as to operate in the TEM_{00} mode. This is done at least partially by ensuring that the ratio of the length of the tube to its diameter is such that the TEM_{00} mode is favored. Other higher order modes have larger diameters and are cutoff by the narrow aperture defined by the plasma tube. So helium-neon lasers typically emit TEM_{00} beams, with diameter about a millimeter and divergence about a milliradian.

8.3 Liquid Dye Lasers

Liquids have useful advantages in relation to both solid and gas laser media. Solids are **very** difficult to prepare with the requisite degree of optical homogeneity and they may suffer permanent damage if overheated. Gases do not suffer from these difficulties but have a much smaller density of active atoms. Several different liquid lasers have been developed but the most important is the dye laser. The important feature that dye lasers offer is tunability, i.e. it has the advantage that it can be tuned over a significant wavelength range, from near I.R. to near U.V. This is extremely useful in many applications such as spectroscopy and the study of chemical reactions.

The active medium is an organic dye dissolved in a solvent. The dye materials are relatively complex organic molecules, with molecular weights of several hundred. For example, Rhodamine 6G is one of the important dye material to be used for lasing action. It contains several benzene rings and has the chemical formula $C_{26}H_{27}N_2O_3Cl$ with a molecular weight of around 450. The dye materials are dissolved in organic solvents, commonly methyl alcohol. Thus the active martial for dye lasers is a liquid.

8.3.1 Energy Level Diagram

Optical pumping is used for dye materials. When the dye is excited by short wavelength light, it emits radiation at a longer wavelength i.e., it fluoresces. The energy difference between the absorbed and emitted photons ultimately appears as heat. Typical absorption and emission spectra are shown in Figure 8.4**.** The broad fluorescence spectrum can be explained by the energy level diagram of a typical dye molecule.

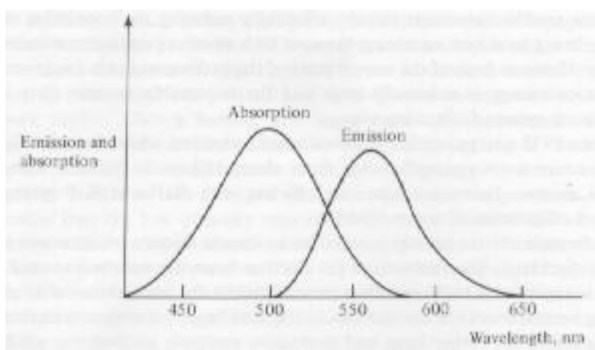


Figure 8.4 Absorption and emission spectra of a typical dye laser.

A typical energy level diagram of a dye material is shown in Figure 8.5. This figure shows

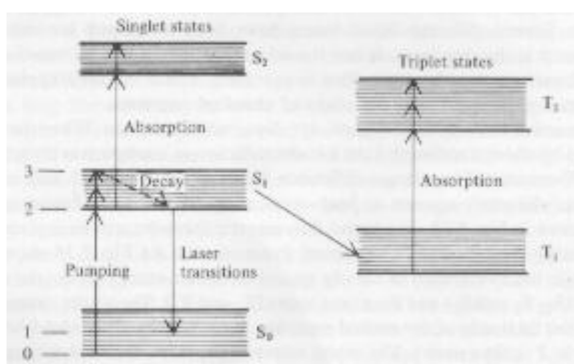


Figure 8.5 Energy level scheme for a dye molecule.

that the dye molecules have two groups of closely spaced electronic energy levels, the singlet states (S_0, S_1 and S_2) and the triplet states (T_1 and T_2). Each electronic energy level is broadened into a near continuum of levels by the effects of vibration and rotation of the dye molecule and also by the effects of the solvent molecules.

When the dye material is irradiated with light whose wavelength corresponds to the energy difference between S_0 and S_1, some of the ground state molecules are raised to sub-levels of S_1. The molecules from sub-levels of state S_1 decay in a very short time to the lowest lying sub-levels of the state S_1. This decay occurs by non-radiative transitions which produces heat. The upper sub-levels of the state S_0 are initially empty. Thus population inversion is achieved between S_1 and the upper sublevels of S_0. Since there are many such rotational/vibrational levels within S_0 and S_1 there are many transitions resulting in an emission line, which is very broad. This leads to the possibility of tuning dye lasers. As the termination of the laser transition in S_0, is at energy much larger than kT above the bottom of S_0, the dye laser is a four level system and threshold is reached at a very small population inversion.

8.3.1.1 Effects of the presence of T_1 and T_2 States

Although triplet states are not directly involved in the laser action, they have a profound effect, as there is a small probability of a transition $S_1 \rightarrow T_1$, even though this is forbidden by quantum mechanical selection rules. However, when dye molecules are excited to the state S_1, some of them are capable of reaching to state T_1 because of collisions. Since the transition $T_1 \rightarrow S_0$ is also forbidden, molecules pile up in T_1. This process steals the molecules that could otherwise be operative in laser operation. The transition $T_1 \rightarrow T_2$ is allowed, and unfortunately the range of frequencies required for this transition is almost exactly the same as the laser transition frequencies. Thus once a significant number of molecules have made the $S_1 \rightarrow T_1$ transition, absorption of $T_1 \rightarrow T_2$ reduces the gain and may stop the laser action. For this reason most dye lasers operate in short pulses, shorter than the time taken for T_1 to acquire a significant population, which is typically 1 μs. For long pulse or cw operation the population in T_1 will build up to equilibrium values, in which absorption is high and becomes the ultimate limitation on the efficiency of the laser.

Continuous dye lasers have been produced by rapidly circulating the dye molecules through a continuous pumping beam. The result is a dye laser, which has a continuous output. However, each molecule of the dye is irradiated by a brief pulse of pumping light as it passes through the pump beam. Then as molecules accumulate in state T_1 in a particular volume of fluid, that volume is circulated out of the laser cavity and is replaced by fresh fluid. The dye can be circulated by a pump. The circulation time is long enough so that the state T_1 can decay before a particular molecule return to the laser region.

8.3.2 Pumping Sources

Dye lasers are optically pumped, the pumping source having a wavelength slightly less than that of the laser output. Commercial pumping methods include flash lamps, nitrogen lasers, excimer lasers, solid state lasers and ion (Ar^+ or Kr^+) lasers. Out of these sources, Ar^+ laser (as well as Kr^+ laser) is a continuous source of light, all other pumping sources are generally operate in pulsed mode and produce a pulsed output from the dye laser. The choice of the pump source depends on the absorption spectrum of the dye being used and the type of output desired. In practice, lasers are used as the pumping sources for dye lasers. We shall discuss two configurations which have been employed. One geometry uses a pulsed nitrogen laser, shown in Figure 8.6. The population of T_1 builds up in about 1 μs, whereas the nitrogen laser operates only in very short pulses, with duration of the order of 100 ns. Thus, the problem with population of state T_1 is eliminated.

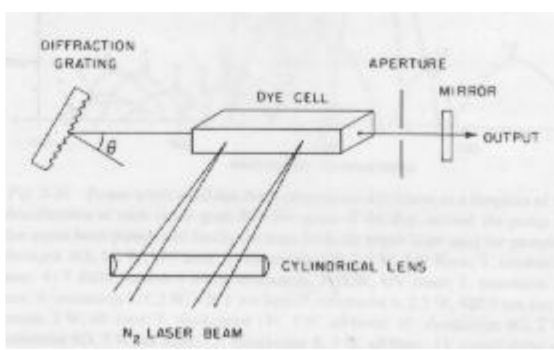


Figure 8.6 Schematic diagram of a puled dye laser excited by a Nitrogen laser beam, with tuning by a diffraction grating.

In Figure 8.6, the wavelength selecting element is a diffraction grating which serves as one of the mirrors. The diffraction grating obeys the equation

$$n\lambda = 2d \sin\theta \qquad (8.2)$$

where n is a small integer, λ the wavelength, d the spacing between rulings, and θ the angle between the light and the normal to the grating. Light of a particular wavelength which will be reflected back into the cavity can contribute to laser action. Thus variation of θ can pick a particular value of wavelength. Therefore, rotation of the grating can serve to vary the wavelength of laser operation.

A second design which represents a continuous dye laser pumped by an argon laser is shown in Figure 8.7. The argon laser beam is focused to a small spot and the dye flows through the

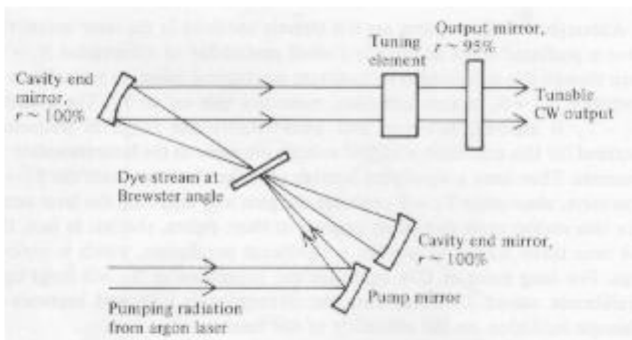


Figure 8.7 Schematic diagram of a tunable CW dye laser. The dye stream flow is perpendicular to the page.

dye laser cavity. A typical velocity for the dye flow is around 10m/s. If the pump beam is focused to a diameter around 10μm, the effective pumping pulse as the dye molecules pass through the pumping beam is around 1μsec. The laser efficiency and output powers are improved if the fluid flow is made as fast as possible and the diameter of the pump beam is made as small as possible. The dyes are forced to flow in a high jet, which is tilted at Brewster's angle to the incoming pump light. The curved mirror, shown in Figure 8.7, is used to focus the pump light. A wavelength-selecting filter is used as the tuning element. By rotating the filter, light of one particular wavelength will be able to reach the mirror and be reflected back into the laser cavity for further stimulated emission.

8.3.3 Tuning Ranges

The entire ranges of wavelengths (from near I.R. to U.V.) can't be achieved using a single dye material. Each dye material has a certain range of wavelength emission. A typical tuning range for one particular dye is several tens of nanometer. When the end of this tuning range is reached, one may physically change the dye material that is being used. This can be done by physically lifting out the reservoir of the dye and replacing it with a reservoir containing a different dye. In this way tuning across the entire visible spectrum may be obtained. This is illustrated in Figure 8.8, which shows the available power levels which may

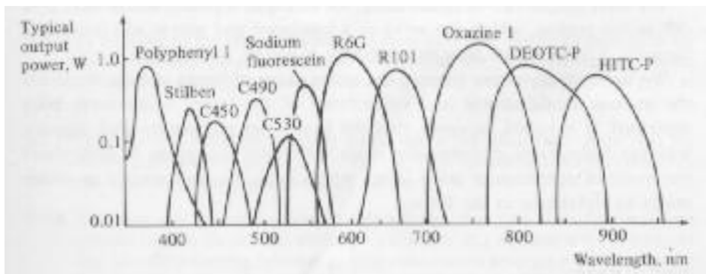


Figure 8.8 Relative outputs of some common dyes pumped by ion lasers.

be obtained in a continuous flowing dye laser pumped by an argon laser. Thus one can tune between approximately 570 to 650 nm with rhodamine 6G. This dye has an efficiency of 20%. When the end of the tuning range of this dye is reached, the rhodamine 6G is replaced with another dye and one may proceed further to the red. When the end of the tuning range of this dye is reached, another dye may be used, and so on. In this way, dye lasers can be tuned throughout the entire visible spectrum.

8.4 Solid-State Lasers

A solid-state laser is one in which the atoms that emit light are fixed within a crystal or a glassy material. The materials used for solid-state lasers are electrically non-conductive, and must be excited optically from an external source.

These lasers are rugged, easy to maintain and capable of generating high peak powers. Typical examples are Ruby Laser, Nd:YAG Laser, Nd-Glass Laser.

We will discuss Nd-YAG Laser.

8.4.1 The Nd:YAG Laser

Neodymium:YAG lasers are the most popular type of solid state lasers. These are four-level laser systems. They can be pumped by flash lamp, arc lamp or semiconductor lasers. These lasers can be operated either cw or pulsed. In Nd-YAG laser, Neodymium serves as active medium, YAG (Ytterium Aluminium Garnet) is the host. The Nd:YAG laser operates at wavelength of 1.064 μm (near I.R.). Its 2nd and 3rd harmonic can shift the output wavelength from the near I.R. into the visible or ultraviolet. Neodymium lasers can generate continuous beam of a few milliwatts to over a ten of watts, or pulsed beam with average powers in the kilowatt range or in short pulses with peak powers in the terra-watt range.

Neodymium lasers have found many applications in research, industry, medicine, military equipments etc. Commercial types range from small-diode pumped models emitting several milliwatts continuous light, to materials working system generating 1 kilowatt. Research versions cover an even broader range, from tiny fiber lasers to building sized lasers used for fusion research in advanced laboratories.

8.4.1.1 Yttrium Aluminum Garnet

Yttrium Aluminum Garnet is probably the most commonly available crystalline laser host. It is a hard isotropic crystal. Its chemical formula is $Y_3Al_5O_{12}$. Its crystalline structure is similar to that of garnet. In YAG, some of the Y^{+3} ions are replaced by Nd^{+3} ions. Typical doping level of neodymium is about 1% (by weight). Higher doping leads to strained crystal since the radius of the Nd^{+3} ion is ~ 14% larger than that of the Y^{+3} ion. This doping level makes the YAG crystal (which is otherwise transparent) to appear pale purple in color because of the Nd^{+3} absorption bands in the red. With this doping level, the neodymium concentration is of the order of 10^{20} atoms per cubic centimeter; considered optimum for lasing action. The active Nd^{+3} ion surrounds itself with several oxygen atoms that largely shield it from its surroundings.

Typically YAG rods are 3 to 9 mm in diameter and up to 10 cm long. The crystal has good thermal, optical and mechanical properties. It offers low value of threshold and high values of gain. Its hardness is 8.5 on Moh scale. It has high thermal conductivity; over ten times that of glass. These desirable properties allow the Nd-YAG rod to produce a good quality laser beam, something difficult at room temperature for most other solid state laser materials.

8.4.1.2 Pumping Mechanisms

The thermal and optical properties of Nd:YAG let it to be pumped either continuously or pulsed. Medium pressure (500 — 1500 torr) Xe-lamps and high pressure (4 — 6 atm.) Kr-lamps are used for the pulsed and continuous wave operations respectively. Diode lasers can also be used for pumping of solid state lasers.

Lamp Pumping

Flash lamp can produce high peak powers in pulses, while, arc lamp is operated in a continuous wave mode. Although arc and flash lamp sources offer high intensity but much of their emission is not absorbed by the neodymium ions.

For lamp pumping, hollow reflective cavities such as shown in Figure 8.10, transfer pump light from one or more pump lamps to the laser rod. The first solid state laser was pumped by helical flash lamps placed around the laser rod, as shown in Figure 8.9(a), but this approach

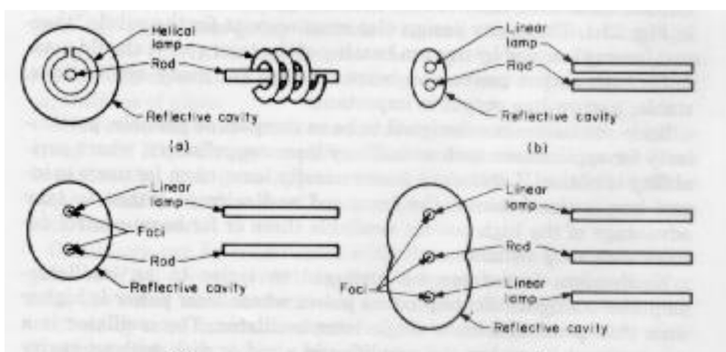


Figure 8.9 Representative cavity configurations for flash lamp pumping (a) helical lamp, (b) closely coupled lamp, (c) elliptical cavity, (d) dual elliptical cavity.

has been supplanted by linear lamps. One modern approach is to place the linear lamp with the laser rod in a closed coupling configuration as shown in Figure 8.9(b). Another is to put the lamp and the laser rod at the two foci of an ellipse, so the geometry efficiently focuses the pump light onto the rod, shown in Figure 8.9(c). Two lamps and a rod can be put into a dual elliptical cavity, shown in Figure 8.9(d), which in cross section looks like two overlapping ellipses with the rod at the common focus.

High power oscillators or amplifiers may be surrounded by arrays of many flash lamps. As the pumping flash last for only a short time (~1 ms) the laser output is in the form of a pulse, which starts about 0.5 ms after the pumping flash starts. This represents the time for the population inversion to build up. Once started, stimulated emission build up rapidly and thus depopulates the upper laser level much faster than the pumping can replace the excited atoms so that laser action momentarily stops until population is achieved again. This process then repeats itself so that the output consists of a large number of spikes of about 1 μs duration with about 1 μs separation.

Diode Laser Pumping

Pumping with diode lasers has become an important technology from mid 1980s; because it offers much higher overall efficiency than lamp pumping. The higher efficiency means that there is less waste heat to remove, allowing smaller packages. GaAlAs semiconductor lasers which emit near 800 nm have been increasingly attractive pump sources.

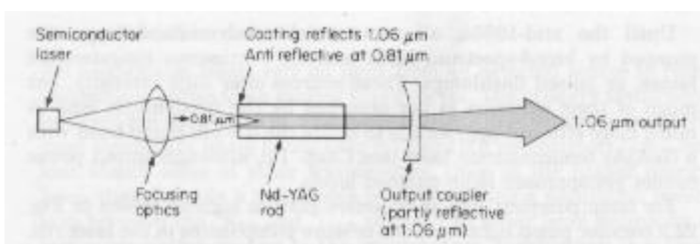


Figure 8.10 End pumping of an Nd-YAG rod with an 810 nm diode laser Diode laser

pumping can be from the end of the rod, as shown in Figure 8.10 or from the side. In end pumping, standard at low power levels, optics focus the rapidly diverging beam from the diode laser to fill as much space as possible in the crystal, which strongly absorbs at the diode laser wavelength near 810 nm. Multiple diode lasers can be arranged along the side of the rod for side pumping for higher powers.

Lamp pumped-neodymium lasers normally operate in multiple longitudinal modes. Diode-pumped-lasers, with much shorter cavities, have broader longitudinal mode spacing and can more easily be limited to oscillate on a single longitudinal mode.

Typical beam divergences from diode-pumped neodymium lasers diameters are 1 to 10 mrad at 1.06 μm. Beam diameters are 0.2 to 2 mm.

8.4.1.3 Energy Level Diagram

Neodymium laser is a four level laser system. In such a laser, the lower level lies far above the ground state and is generally empty. Let n_1 and n_2 denote the number of atoms in the upper and lower laser levels per unit volume, respectively. As $n_1 = 0$, so any population in level 2 gives rise to an inversion with $n > 0$. Such lasers are much more efficient than three level lasers. If level 1 decays to level 0 rapidly enough, i.e., $n_1 \rightarrow 0$ under all conditions and thus four level laser do not absorb (in general) the laser light itself.

When the ion is placed in the host, the crystal field splits some of the energy levels. A simplified energy level scheme for Nd:YAG laser is shown in Figure 8.11. The actual energy

level kinetics are quite complex. The levels shown in Figure 8.11, arise from transitions of the three inner shell 4f electrons of the Nd^{+3} ion in the field of YAG crystal. Since these

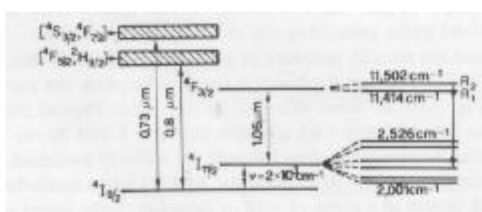


Figure 8.11 Simplified energy levels of Nd:YAG.

electrons are screened by eight outer electrons ($5s^2$ and $5p^6$), the energy levels involved are only weakly influenced by the crystal field and the corresponding transition frequencies are relatively sharp. The two main pump bands occur at 0.73μm and 0.8 μm, respectively; also other higher lying absorption bands play an important role. These bands are coupled by a fast ($\sim 10^{-7}$sec) nonradiative decay to the metastable level $^4F_{3/2}$. So the optically excited neodymium ions quickly decay to this metastable level by releasing their excess energy to the crystalline lattice. From $^4F_{3/2}$, they decay to lower I levels as shown in Figure 8.11. The rate of this decay ($^4F_{3/2} \rightarrow$ I levels) is much slower, i.e., the life time of this state is quite large($\tau \cong$

0.23 ms). The reason for this long life is that the $^4F_{3/2}$-level with a large energy gap with its nearest lower energy level that forbids the non-radiative decay and for non-radiative decay, quantum mechanical selection rules do not allow electric dipole transition to the lower lying levels. This mean that level $^4F_{3/2}$ accumulates a large fraction of the pump energy and is therefore a good candidate as the upper level for laser action. Of the various possible transitions from $^4F_{3/2}$ to lower lying levels, it turns out that the $^4F_{3/2} \rightarrow {}^4I_{11/2}$ is the strongest transition. Level $^4I_{11/2}$ is then coupled by a very fast (ns) non radiative decay to the $^4I_{9/2}$ ground level. The $^4I_{9/2}$ level can thus be considered as a favorable candidate as lower laser level for laser action.

It should also be noted that laser action can be obtained, using suitably dispersive systems in the laser cavity such as those of Figure 8.12 at several other wavelengths corresponding to various $^4F_{3/2} \rightarrow {}^4I_{11/2}$ transitions (λ = 1.05-1.1 μm), various $^4F_{3/2} \rightarrow {}^4I_{13/2}$ transitions (λ = 1.319 μm) and $^4F_{3/2} \rightarrow {}^4I_{9/2}$ transition (λ around 0.95 μm). It is also worth recalling, that the λ = 1.06 μm laser transition is homogeneously broadened at room temperature owing to interaction

with lattice phonons. The corresponding width is $\Delta\nu = 6.5\ cm^{-1}$ (or 195 GHz) at T= 300 K. The long lifetime of the upper laser level (τ = 0.23 ms) also makes Nd:YAG very suitable for Q-switched operation.

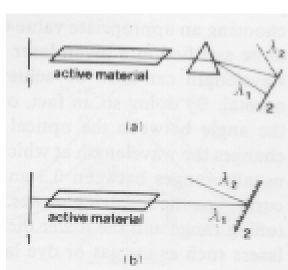


Figure 8.12 Laser tuning using the wavelength dispersive behavior of (a) a prism, (b) a diffraction grating.

8.4.1.4 Nd-Laser Resonators

The gain in neodymium lasers is high enough that they can use either stable or unstable resonators. An unstable resonator has the advantage to utilize the maximum volume of the laser rod for stimulated emission. Close to the laser source, some unstable resonators produce beams with a bright ring around a central point of minimum intensity, which looks like a ring or doughnut in cross-section. However, far from the laser, the hole vanishes to produce a bright central spot. A stable resonator can produce the standard Gaussian TEM_{00} beam with a bright central spot, but it does not extract laser energy from a large fraction of the rod volume. Unstable resonators have been going in popularity because for most applications, output power and energy are more important than near field beam quality.

Three simple resonator designs for lamp-pumped lasers are shown in Figure 8.13. Many lasers have more complex designs, which incorporate interacavity polarizers, beam splitters, or other schemes to couple light out of the cavity.

As solid-state laser cavities are designed to pump and extract maximum energy from the laser medium. Thus the resonator mirror should confine light to oscillate through most of the rod, as shown in Figure 8.13. The cavity should be designed such that it should maximize both output power and beam quality.

Some resonators have a very compact design as possible, particularly for applications such as military laser range finders, where portability is critical. Laboratory lasers usually leave room for users to insert accessories between the laser rod and mirrors.

Solid state laser materials are not always used in the form of cylindrical rods. Disk geometry

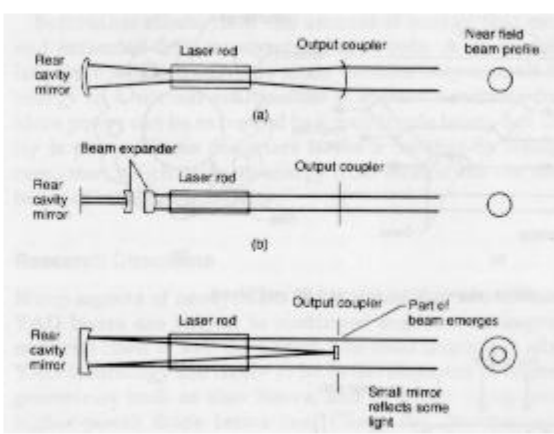


Figure 8.13 Three simple resonant cavity used with lamp pumped Nd:YAG laser: (a) a simple

stable resonator to generate TEM_{00} mode; (b) stable resonator with internal telescope, which helps in generating low divergence beam; (c) unstable resonator with output coupled around a small mirror producing a near field beam with a null at its center. In the far field all three beams have central bright regions.

can also be used, shown in Figure 8.14. Use of thin disks rather than thick chunks of laser glass minimize thermal problems with the material.

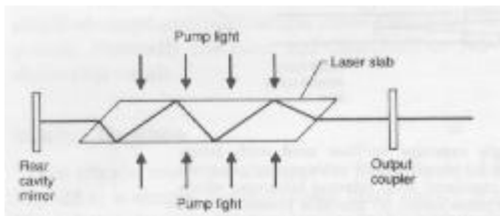


Figure 8.14 A zigzag slab laser.

8.4.1.5 Output Power

A wide range of output power is available from Nd-YAG lasers, depending on the laser configurations, pumping source, and wavelength. Steady or average powers in the kilowatt range can be obtained either from pulsed or continuous wave lasers with the highest power from multimode, oscillator-amplifier configuration and slab designs.

Peak powers available from neodymium lasers are much higher and depend on both the pulse energy and duration. Neodymium lasers can operate in a variety of pulsed modes, depending both on excitation and on control of the energy by Q-switching, modelocking, etc.

Saturation effects limit the amount of energy that can be stored in and extracted from neodymium laser rods. A resonator, which oscillates in a single transverse mode, extracts only a small fraction of the energy in a normal rod because it produces a narrow diameter beam. More power can be extracted in a multimode beam; but the beam quality is poorer. Unstable resonators can be used to extract energy from most of the rod with reasonable beam quality.

Peak power in an emitted pulse of Nd-laser tends to decrease as repetition rate increases beyond a certain point. In general, the higher the peak power, the longer the interval needed between pulses to dissipate excess heat that otherwise might distort the optical properties of the rod and degrade the beam quality of the output beam.

A laser, which is pumped by a flash lamp (with an input energy of about 10 kJ), may produce a laser pulse of about 10 Joules. As the pulse lasts for only 0.5 ms, the peak power is then $2x10^4$ W, this peak power can be greatly increases by Q-switching. The pulses of 10 ns with the peak power up to 10^9 watts can be achieved using Q-switching techniques.

8.4.1.6 Oscillator-Amplifier Configurations

Nd-glass can be arranged in oscillator-amplifier configuration to produce pulses whose laser power is higher than that possible with a single laser oscillator. The oscillator is a conventional laser, but the amplifier is a rod or disk without cavity mirrors, which is separately pumped. A typical oscillator-amplifier configuration is shown in Figure 8.15. The oscillator and amplifier need not use the same host material but the host wavelengths must be close enough that the oscillator's wavelength falls within the gain bandwidth of the laser amplifier. In high energy, low repetition rate systems, oscillators are Nd:YAG but the amplifiers typically are glass rather than crystalline because of (i) the high quality glass laser rods can be made in about any diameter to allow high power with relatively low power density and (ii) Glass is highly resistant to damage from higher power density.

Care must be taken in matching the wavelengths of oscillators and amplifiers made of different materials. Nd:YAG (λ = 1.064 μm) oscillators can be used with silicate glass (λ = 1.062 μm) amplifiers.

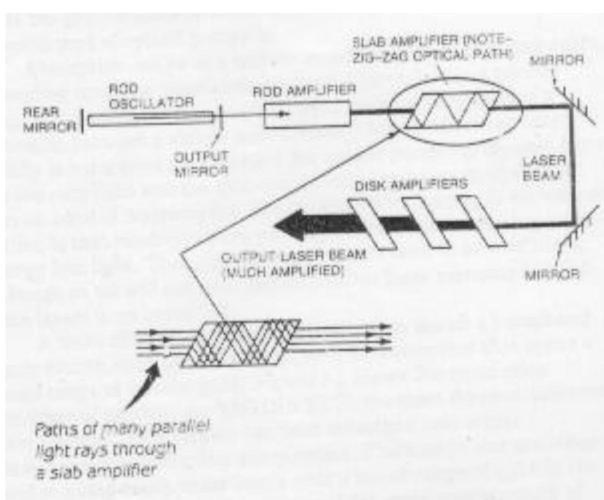


Figure 8.15 Laser oscillator and amplifiers.

Multiple amplifiers may be used where extremely high laser powers are required, such as in laser fusion experiments. A system based on Nd-Glass laser amplifiers, delivering pulses with energy of ~ 100 kJ and peak power of 100 TW, has already been built. These lasers make use of a chain of several Nd-glass amplifiers, one of which consists of Nd-glass disk of ~ 4 cm in thickness and ~ 75 cm in diameter.

8.4.1.7 Efficiency

Overall efficiency of a commercially lamp-pumped neodymium laser, measured as laser power output divided by electrical power in, typically is 0.1 to over 1 percent. Diode-pumped lasers can be considerably more efficient because neodymium absorbs the 0.8-μm pump beam much more efficiently than the broad-band output of a lamp. The optical losses contribute to the low efficiency of lamp-pumped lasers. They account for the following typical fractions of the input energy to a continuous lamp:

- Only half the input energy emerges as pump light at 0.3 to 1.5 μm; the lamp dissipates the other 50 percent as heat.
- Only 8 percent of the original energy is absorbed by the laser rod. The laser cavity absorbs 30 percent, the coolant and flow tubes absorb 7 percent, and the lamp re-absorbs 5 percent.
- Only 2.6 percent of the lamp input energy goes into stimulated emission from the laser rod, 5 percent is lost as heat and 0.4 percent is lost as fluorescence.
- Optical losses reduce the typical overall efficiency to the 1 percent range. The fractions differ somewhat with design details.

8.4.1.8 Cooling

A large amount of heat is dissipated by the flash lamp and consequently the laser rod quickly becomes very hot. To avoid the damage resulting from this, and to allow a reasonable pulse repetition rate, cooling arrangements are necessary for solid-state lasers. For low-power lamp or diode pumped YAG lasers, connective air-cooling is sufficient. However this limits operation to low repetition rates. For high power lasers, it is necessary to use forced air cooling or water cooling.

8.4.1.9 Safety

Pulsed neodymium lasers probably have been responsible for more eye injuries than any other type, both because of their wide spread use and because of the particular hazards they pose.

Although the human eye can not see the 1.06 μm fundamental wavelength of the neodymium laser, that light can pass through the eyeball to damage the retina. The same considerations apply to the 1.3 μm neodymium line. The risk of eye damage due to an unseen beam is increased by the fact that most neodymium lasers used in the laboratory produce short, intense pulses. The best protection against such accidents is safety goggles which effectively block all optical paths to the eye, transmitting most visible light by strongly blocking the near infrared neodymium lines.

Harmonics of neodymium lasers also come in short, intense pulses and present serious eye hazards. The second harmonic at 532 nm is visible green light, and can penetrate the eye. It requires safety goggles, which strongly attenuate the green part of the spectrum.

The 355 nm third harmonic and the 256 nm fourth harmonic pose eye hazards. The fourth harmonic also can cause a sunburn-like effect on the skin. Ultraviolet-blocking safety goggles can provide eye protection against the third and fourth harmonics.

Users should be aware that some light at longer wavelength may remain after harmonic generation unless the optical system is explicitly designed to get rid of it. Some commercial lasers come with optional beam dumps for this purpose. Multiwavelength laser output can pose a severe eye hazard because it is difficult to design safety goggles to block two or more wavelengths.

Continuous wave neodymium lasers pose serious eye hazards, but the power levels are sufficiently low that an instantaneous exposure is less likely to cause permanent damage. Some diode pumped lasers produce continuous beams on the order of a milliwatt at the second harmonic wavelength, but most neodymium lasers generate considerably high powers, eye protection is necessary when working with any neodymium laser.

Lamp pumping poses two additional hazards. All lamps require high drive voltages, posing serious electrical hazards if any high voltages are exposed.

We have by no mean covered the entire range of lasers or fully discussed the various modification and refinements of lasers which have been described. It is hoped, however, that the basic laser physics covered together with the descriptions of some laser types will enable the reader to understand the mode of operation of other lasers which might be encountered or which might be developed in the future.

Problems

8.1 Which major components are required for a laser system? Discuss the function of each component.

8.2 Discuss in detail the pumping mechanism for a He-Ne laser.

8.3 Explain how a He-Ne laser can be operated in green region?

8.4 With the help of a graph, discuss the effect of changing transmitivity of out-put mirror on the output power of a He-Ne laser.

8.5 Calculate the length of the cavity for a He-Ne laser, which is to be operated in a single longitudinal mode.

8.6 With the help of energy level diagram of a liquid dye laser, justify the statement: “Dye lasers are tunable”.

8.7 Describe the effects of T_1 and T_2 states on lasing action in a dye laser. How these effects are eliminated or minimized?

8.8 Give a comprehensive comparison of different optical pumping sources which can be used for a solid-state laser.

8.9 What is oscillator amplifier configuration? Give a schematic diagram of this configuration for Nd-YAG laser. Explain it in detail.

8.10 Discuss the factors, which cause inefficiency of Nd-YAG lasers.

Books for further reading

A. E. Seigman, *Lasers*, (University Science Books, California, 1986).

O. Svelto, *Principles of Lasers*, 3rd ed. (Plenum Press, New York, 1989).

P. W. Milonni and J. H. Eberly, *Lasers,* (John Wiley & Sons, New York, 1991).

J. Wilson and J.F.B. Hawkes, *Optoelectronics: An Introduction*, (Prentice-Hall, India, 1996).

J. Hecht, *Laser Guide Book*, 2nd ed. (McGraw-Hill, New York, 1992).

B. A. Lengyel, *Introduction to Laser Physics*, (John-Wiley & Sons, New York, 1966).

K. Shimoda, *Introduction to Laser Physics* , (Springer-Verlag, New York, 1984).